Einstein's equations with the Robertson-Walker Metric and perfect fluid energy-momentum tensor

$=$

$$\left(\frac{\dot{a}(t)}{a(t)}\right)^2 = \frac{8\pi G}{3}\rho(t) - \frac{kc^2}{a(t)^2}$$

$$\dot{\rho}(t) + 3\frac{\dot{a}(t)}{a(t)}\left(\rho(t) + \frac{p(t)}{c^2}\right) = 0$$

Einstein's equations with the Robertson-Walker Metric and perfect fluid energy-momentum tensor

=

$$\left(\frac{\dot{a}(t)}{a(t)}\right)^2 = \frac{8\pi G}{3}\rho(t) - \frac{kc^2}{a(t)^2}$$

$$H(t) \equiv \frac{\dot{a}(t)}{a(t)}$$

↓

$$H(t)^2 = \frac{8\pi G}{3}\rho(t) - \frac{kc^2}{a(t)^2}$$

$$\dot{\rho}(t) + 3\frac{\dot{a}(t)}{a(t)}\left(\rho(t) + \frac{p(t)}{c^2}\right) = 0$$

Critical density…?

$$H(t)^2 = \frac{8\pi G}{3}\rho(t) - \frac{kc^2}{a(t)^2}$$

$$\rho_{\mathrm{crit}}(t) = \frac{3H(t)^2}{8\pi G}$$

k=0 gives critical density

$$\Omega_i(t) = \frac{\rho_i(t)}{\rho_{\mathrm{crit}(t)}} = \rho_i(t)\frac{8\pi G}{3H(t)^2}$$

Now how do we go further…?

$$\dot{\rho}(t) + 3\frac{\dot{a}(t)}{a(t)}\left(\rho(t) + \frac{p(t)}{c^2}\right) = 0$$

- Need equation of state

$$\dot{\rho}_i(t) + 3\frac{\dot{a}(t)}{a(t)}\left(\rho_i(t) + \frac{p_i(t)}{c^2}\right) = 0$$

$$p/c^2 = \omega\rho$$

Evolution of energy density as a function of a

$$\dot{\rho_i}(t) + 3\frac{\dot{a}(t)}{a(t)}\rho_i(t)\,(1+\omega) = 0$$

$$\frac{\mathrm{d}\rho_i}{\rho_i} = -3(1+\omega)\frac{\mathrm{d}a}{a}$$

$$\rho_i(a) = \rho_{i,0}\; a^{-3(1+\omega)}$$

Equation of state for:

$$\rho_i(a) = \rho_{i,0}\, a^{-3(1+\omega)}$$

- cold matter $\omega = 0$ $\rho_{\rm m} \propto 1/a^3$
- radiation $\omega = \frac{1}{3}$ $\rho_{\rm r} \propto 1/a^4$
- cosmological constant $\omega = -1$ $\rho_\Lambda \propto \text{const}$

$$\rho c^2 = \frac{g}{(2\pi)^3} \int E(\mathbf{p}) f(\mathbf{p}) d^3|p|$$

$$p = \frac{g}{(2\pi)^3} \int \frac{|\mathbf{p}|^2}{3E} f(\mathbf{p}) d^3|p|$$

$$f(\mathbf{p}) = (\exp((E-\mu)/T) \pm 1)^{-1}$$

$$E^2 = m^2c^4 + p^2c^2$$

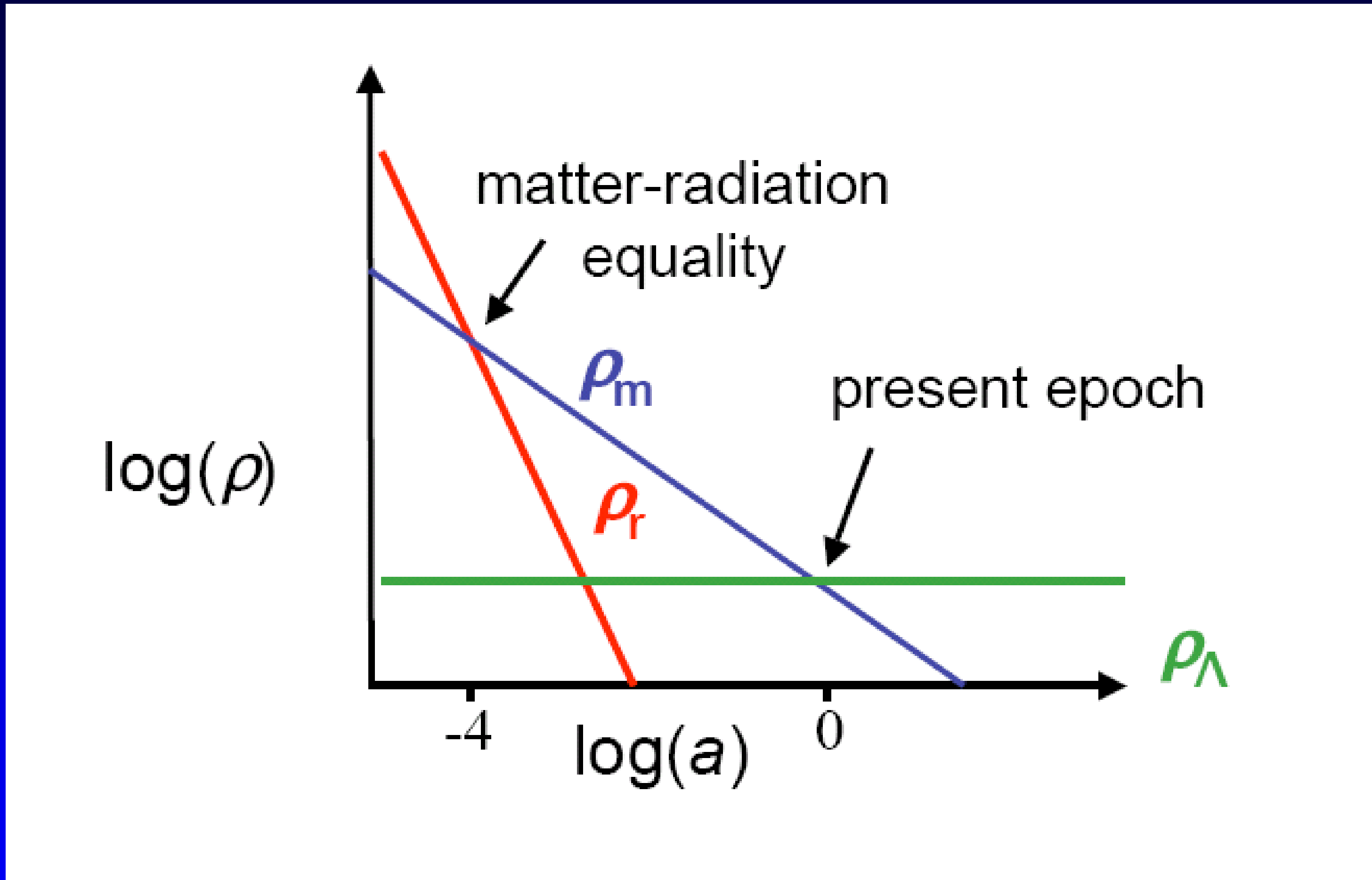
$$\rho c^2 = \frac{g}{2\pi^2} \int_m^\infty \frac{(E^2 - m^2c^4)^{1/2}}{\exp((E-\mu)/T) \pm 1} E^2 dE$$

$$p = \frac{g}{6\pi^2} \int_m^\infty \frac{(E^2 - m^2c^4)^{3/2}}{\exp((E-\mu)/T) \pm 1} dE.$$

relativistic limit

$$E^2 - m^2c^4 \sim E^2$$

$$p = \rho c^2/3.$$



$\omega(t)$

time dependent omega

Time dependence of scale factor *a*

- In the case of k=0 Friedmann equation becomes

$$\dot{a}^2 = \frac{8\pi G\rho_0}{3} a^{-(1+3\omega)}$$

$$a(t) = \left(\frac{t}{t_0}\right)^{2/3+3\omega}$$

$\omega \neq -1$

radiation dominated

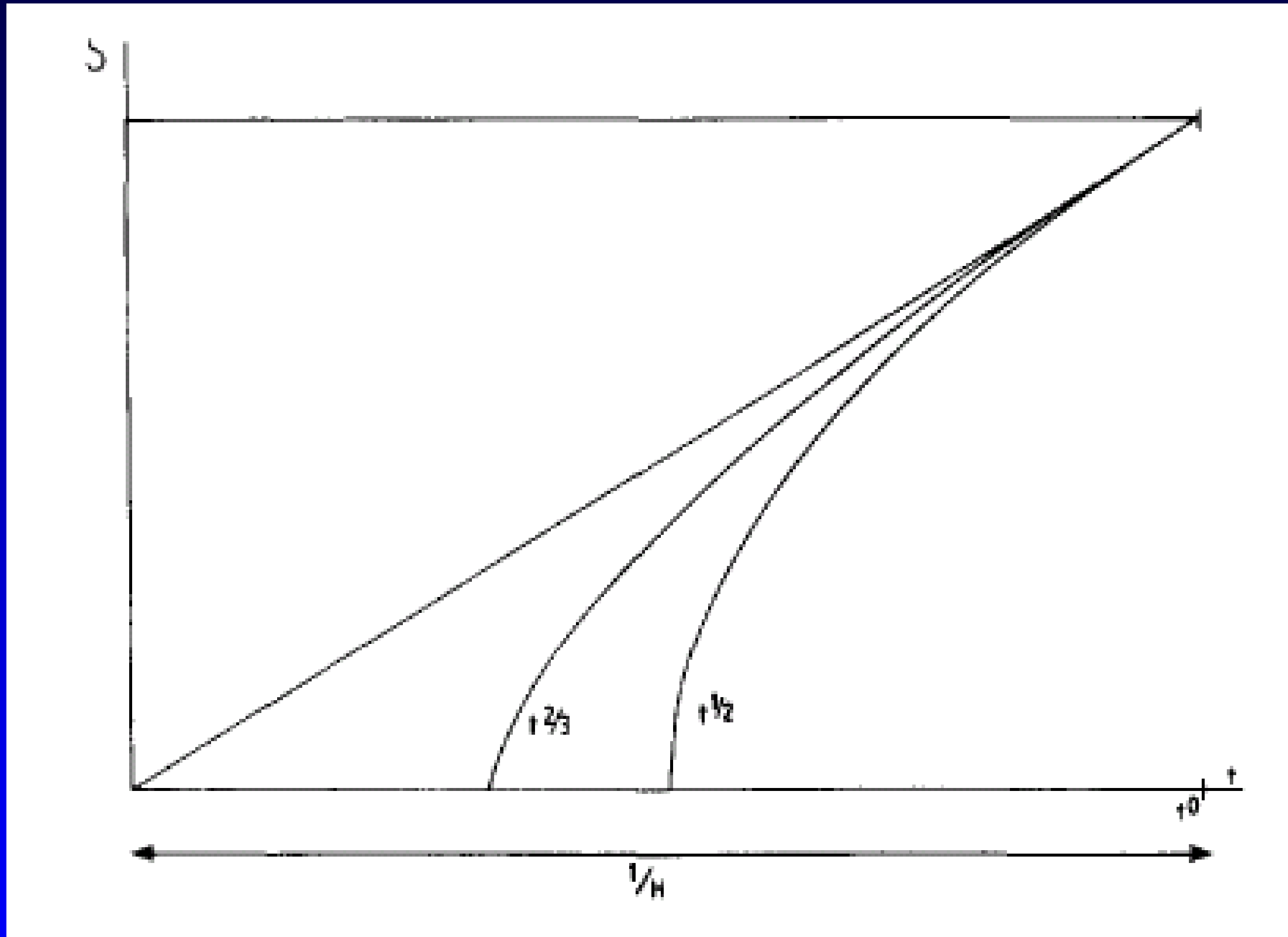
$$a(t) = \left(\frac{t}{t_0}\right)^{1/2}$$

matter dominated

$$a(t) = \left(\frac{t}{t_0}\right)^{2/3}$$

cosmological constant

$$a(t) \propto \exp Ht$$


S
t^2/3
t^1/2
t0
t
1/H

In general can write

$$\frac{H^2}{H_0^2} = \frac{\Omega_{r,0}}{a^4} + \frac{\Omega_{m,0}}{a^3} + \Omega_{\Lambda,0} + \frac{1-\Omega_0}{a^2}$$

and solve numerically

Distances…

- proper distance to galaxy from which light is detected

$$d_p = c\int_{t_e}^{t_0} \frac{\mathrm{d}t}{a(t)}$$

$$\mathrm{d}s^2 = c^2\mathrm{d}t^2 - a^2(t)\left[\mathrm{d}r^2 + \left.\begin{matrix} R^2\sin^2(r/R) \\ r^2 \\ R^2\sinh^2(r/R)\end{matrix}\right\}(\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\phi^2)\right]$$

Distances…

- proper distance to galaxy from which light is detected

$$c\int_{t_e}^{t_0} \frac{\mathrm{d}t}{a(t)} = \int_0^{d_p} \frac{\mathrm{d}r}{1-kr^2}$$

$$\mathrm{d}s^2 = g_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu = c^2\mathrm{d}t^2 - a^2(t)\left[\frac{\mathrm{d}r^2}{1-kr^2} + r^2(\mathrm{d}\theta^2 + \sin^2\theta \mathrm{d}\phi^2)\right]$$

Proper distance for matter domination

$$a_0 d_p = \frac{c}{H_0\, q_0^2\, (1+z)} \left[z q_0 + (q_0 - 1)\left(\sqrt{2 q_0 z + 1} - 1\right)\right]$$

$$q_0 = \frac{-\ddot{a}_0}{a_0\, H_0^2}$$

Luminosity Distance

- In a static Euclidean space the flux detected at a distance d and the luminosity are related by

$$f = \frac{L}{4\pi d^2}$$

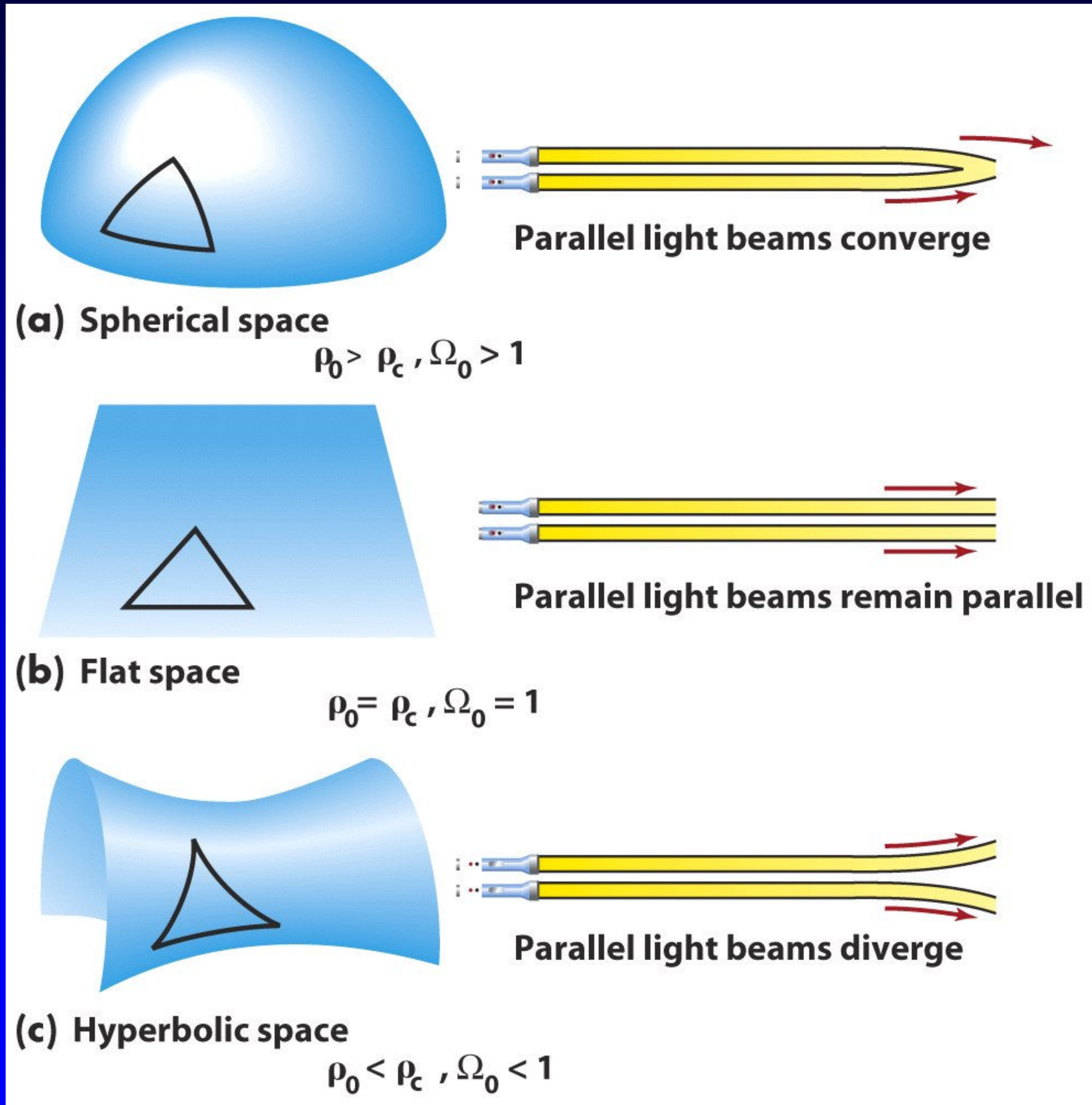
Luminosity Distance

- We see light that was emitted from (r, θ, φ)
- The light is now spread over a sphere of proper radius

$$d_p(t_0) = r$$

and proper surface area

$$A_p(t_0) = 4\pi S_k(r)^2 = 4\pi \begin{cases} R^2 \sin^2(r/R) \\ r^2 \\ R^2 \sinh^2(r/R) \end{cases}$$


Parallel light beams converge
(a) Spherical space
ρ0 > ρc , Ω0 > 1
Parallel light beams remain parallel
(b) Flat space
ρ0 = ρc , Ω0 = 1
Parallel light beams diverge
(c) Hyperbolic space
ρ0 < ρc , Ω0 < 1

Luminosity Distance

- In an expanding universe with curvature *k*

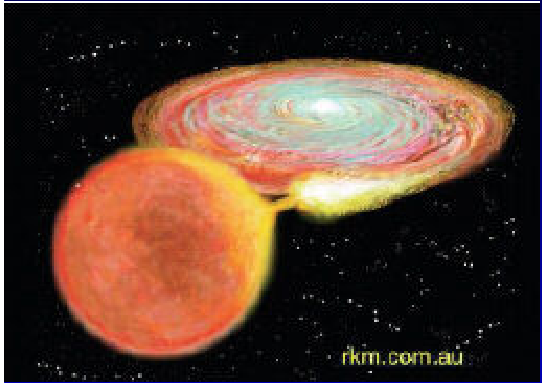
$$f = \frac{L}{4\pi S_k(r)^2(1+z)^2}$$

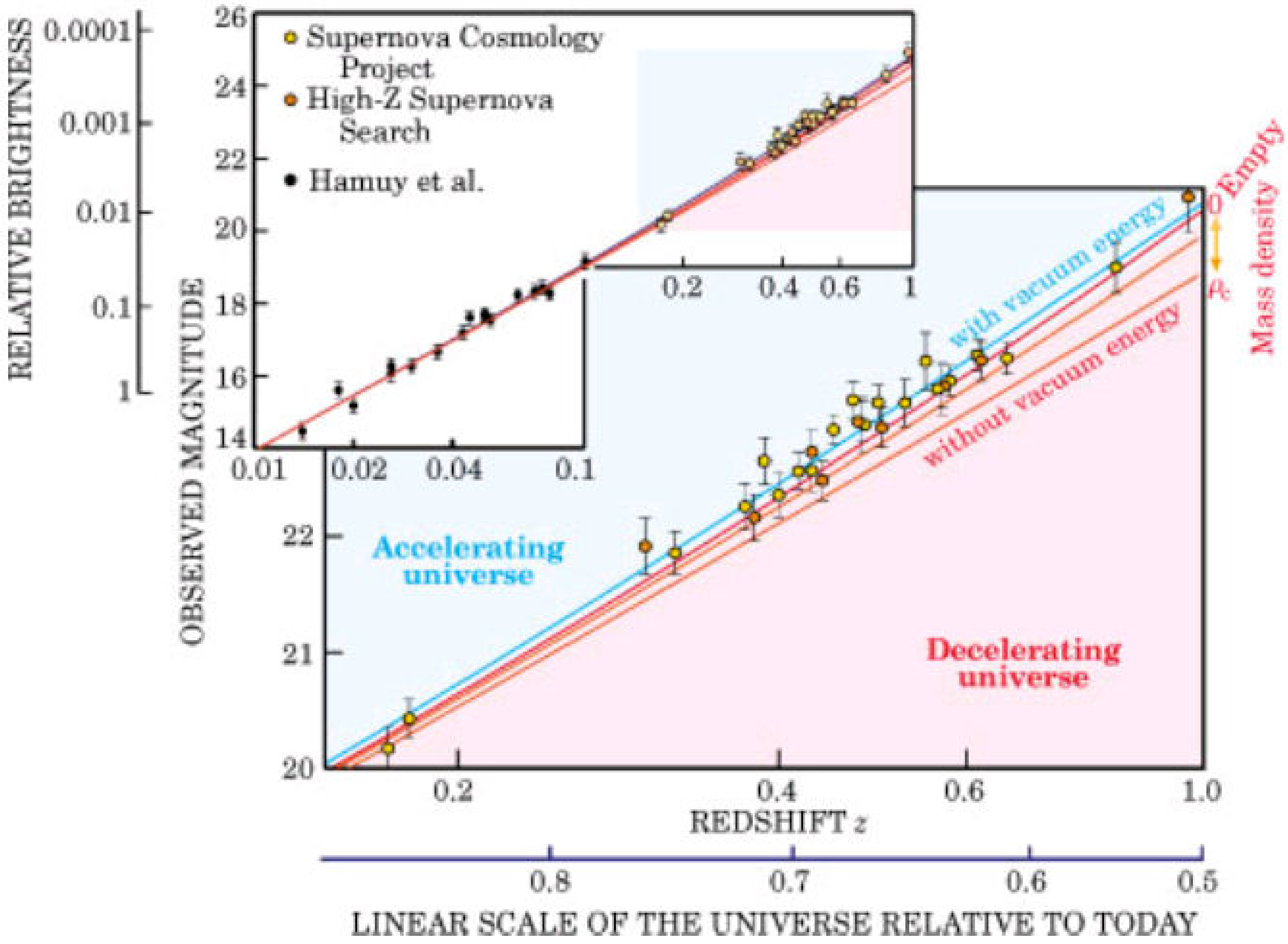
- What is the origin of the two factors of (1+z) ?

k=0

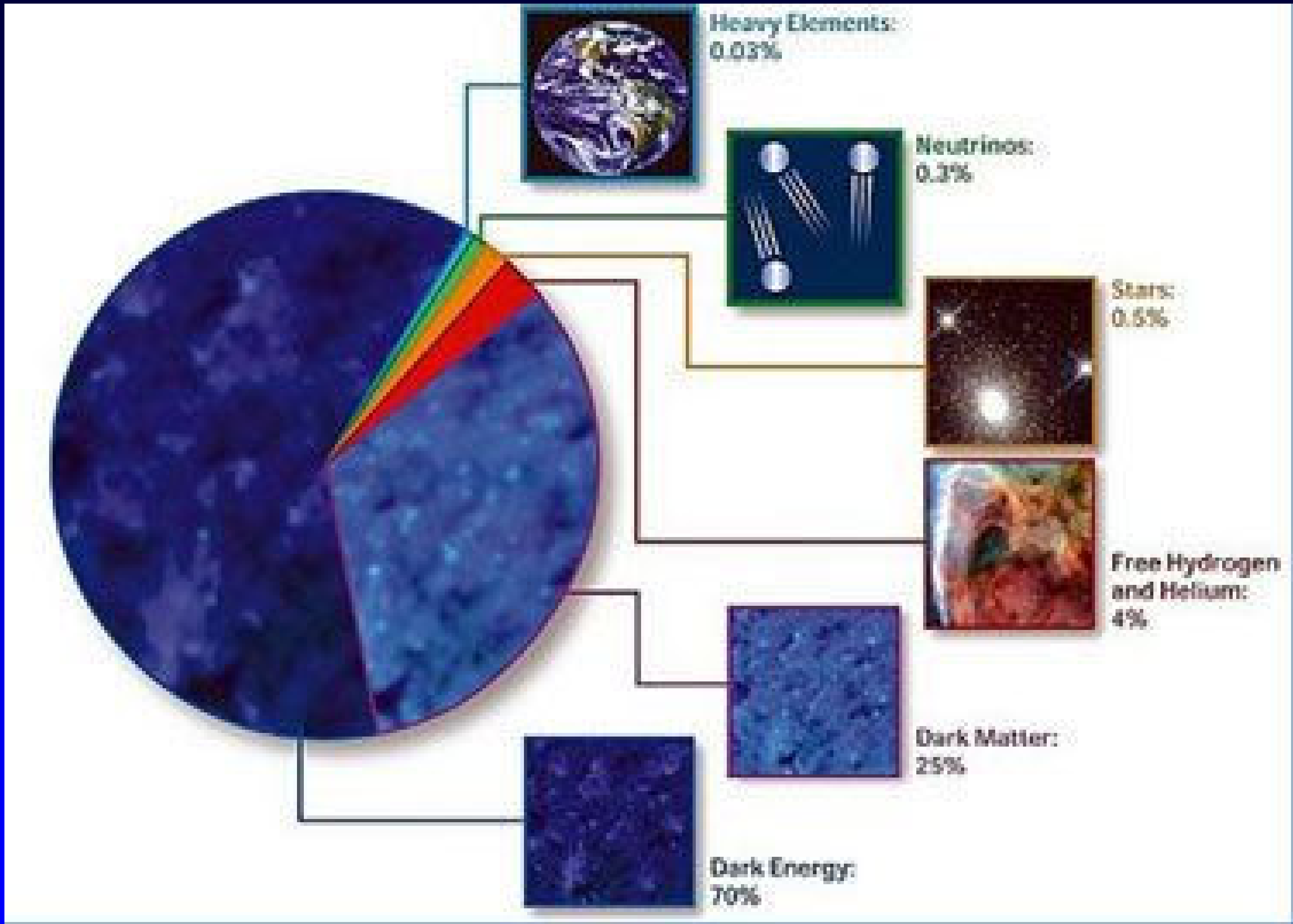
$$d_L = d_p(t_0)(1+z)$$

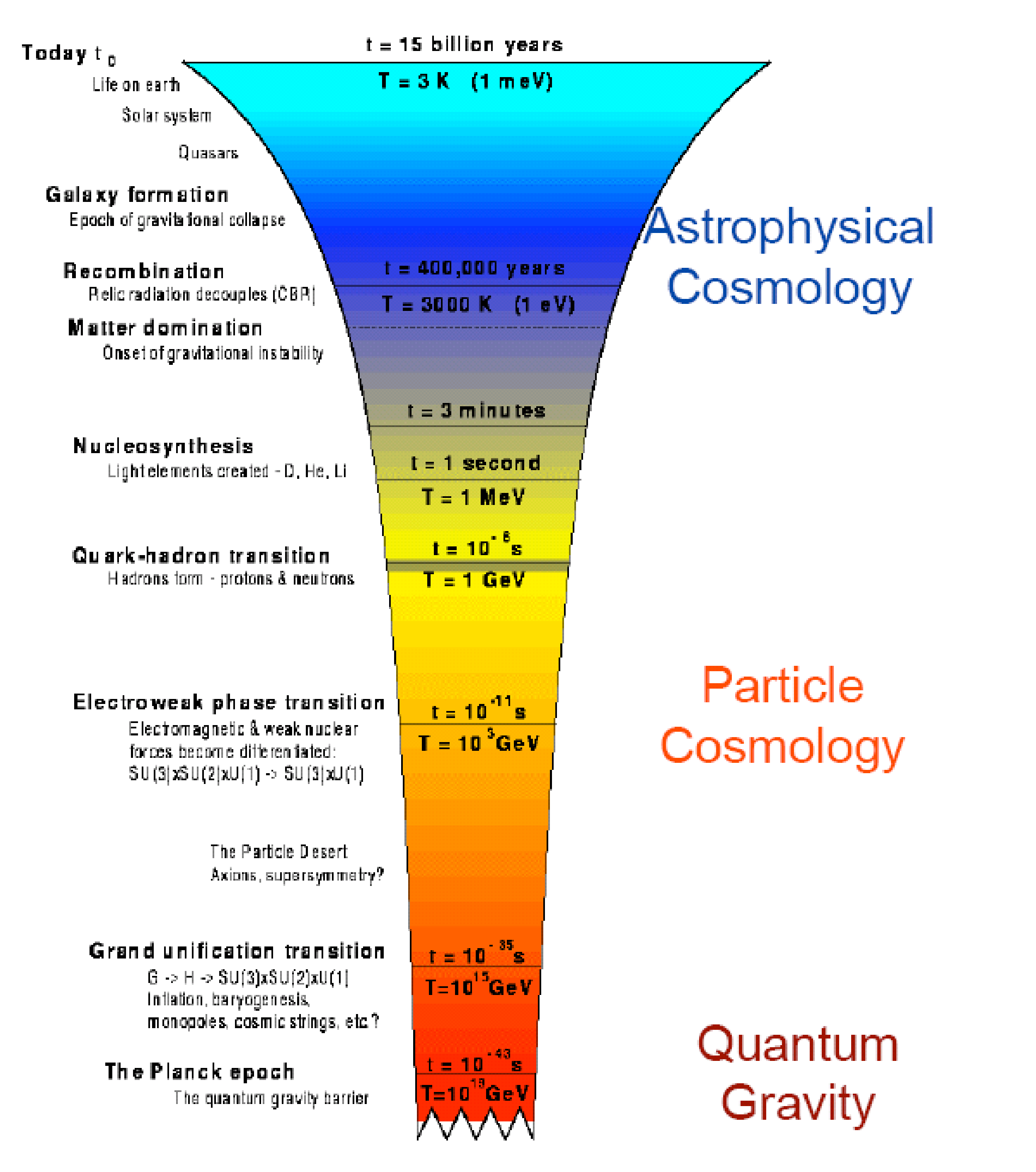
Standard Candles – Type 1a Supernovae





$$M - m = -5\log_{10}\left(\frac{d_L}{10pc}\right)$$


Heavy Elements:
0.03%
Neutrinos:
0.3%
Stars:
0.5%
Free Hydrogen
and Helium:
4%
Dark Matter:
25%
Dark Energy:
70%



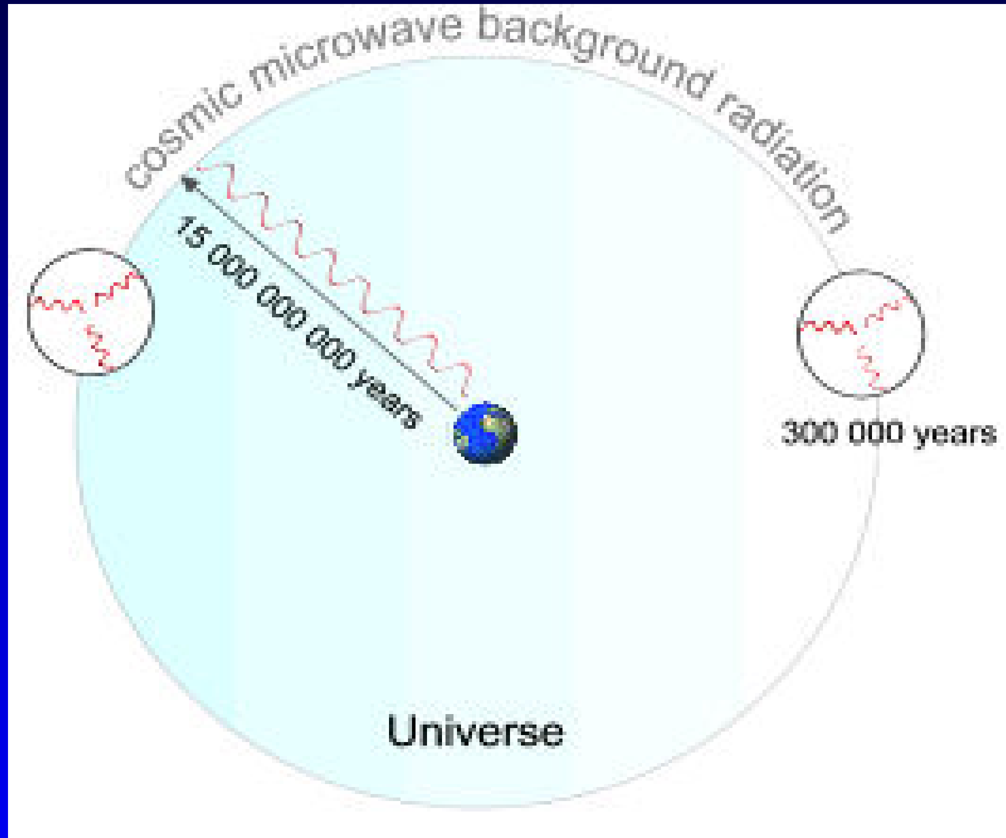
Today t_0
Life on earth
Solar system
Quasars
Galaxy formation
Epoch of gravitational collapse
Recombination
Relic radiation decouples (CBR)
Matter domination
Onset of gravitational instability
Nucleosynthesis
Light elements created - D, He, Li
Quark-hadron transition
Hadrons form - protons & neutrons
Electroweak phase transition
Electromagnetic & weak nuclear forces become differentiated: SU(3)xSU(2)xU(1) -> SU(3)xU(1)
The Particle Desert
Axions, supersymmetry?
Grand unification transition
G -> H -> SU(3)xSU(2)xU(1)
Inflation, baryogenesis, monopoles, cosmic strings, etc.?
The Planck epoch
The quantum gravity barrier
t = 15 billion years
T = 3 K (1 meV)
t = 400,000 years
T = 3000 K (1 eV)
t = 3 minutes
t = 1 second
T = 1 MeV
$t = 10^{-6}$ s
T = 1 GeV
$t = 10^{-11}$ s
$T = 10^{3}$ GeV
$t = 10^{-35}$ s
$T = 10^{15}$ GeV
$t = 10^{-43}$ s
$T = 10^{19}$ GeV
Astrophysical Cosmology
Particle Cosmology
Quantum Gravity

Outstanding issues in the Standard Model of Cosmology…?

- Problems which Inflation solves
  - Horizon problem
  - Flatness problem
  - Monopole problem
  - Origin of structure seeds
- Baryon Asymmetry
- Nature of dark matter and dark energy

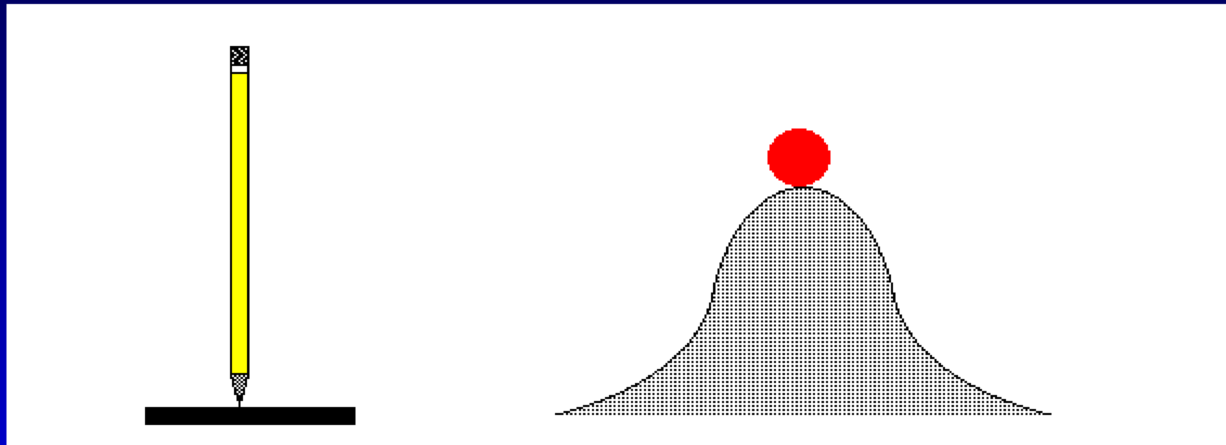
Problems with the Big Bang:
Horizon Problem

- The particle horizon at the time of decoupling was around 0.4 Mpc
- This is around 2 ° on the sky



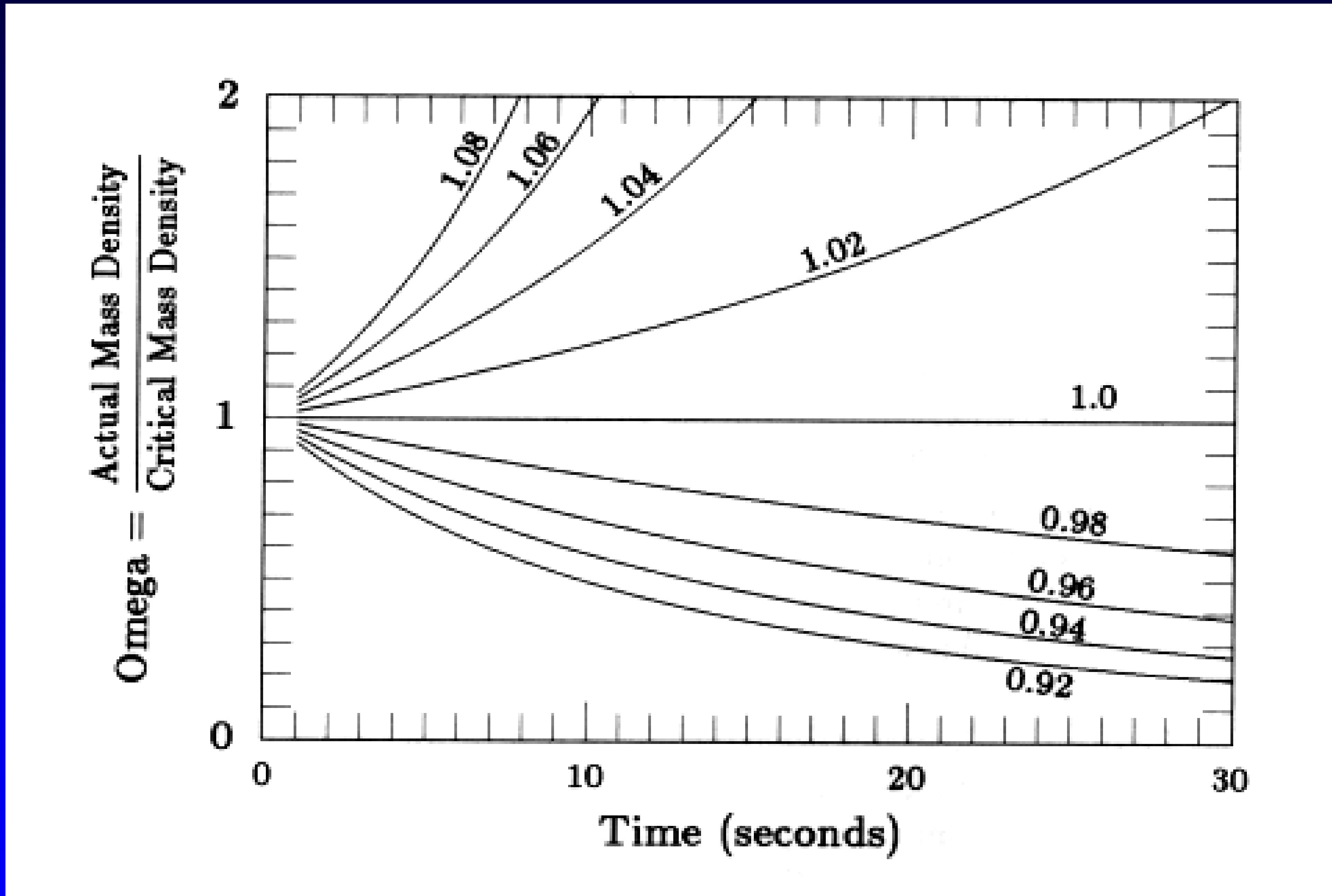
- How can 50, 000 regions which weren’t causally connected at the time of last scattering all have the same temperature (to one part in 100 000)?

Problems with the Big Bang:
Flatness problem



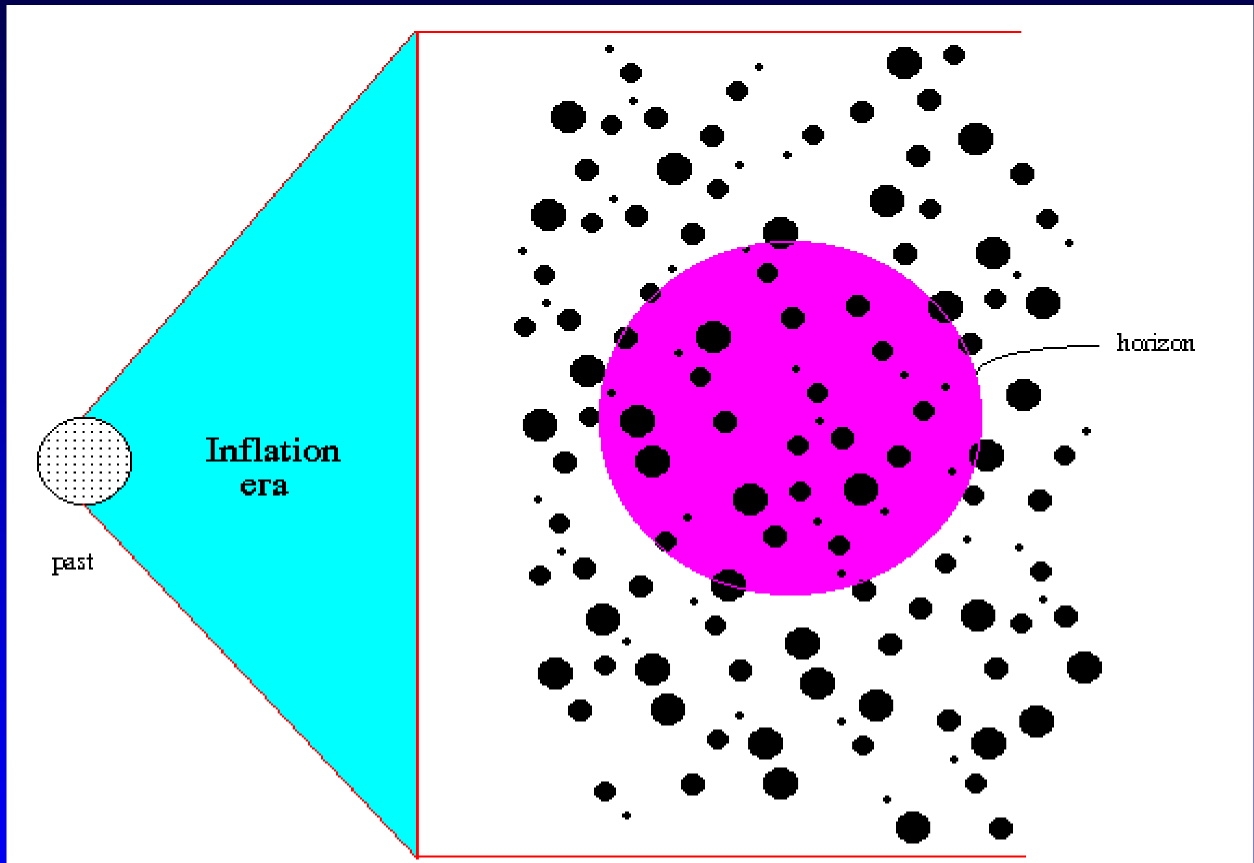
- $\Omega = 1$ is an unstable point for the evolution
- For Ω to be between 0.1 and 2 now it must have been extremely close to 1 in the past

Problems with the Big Bang: *Flatness problem*



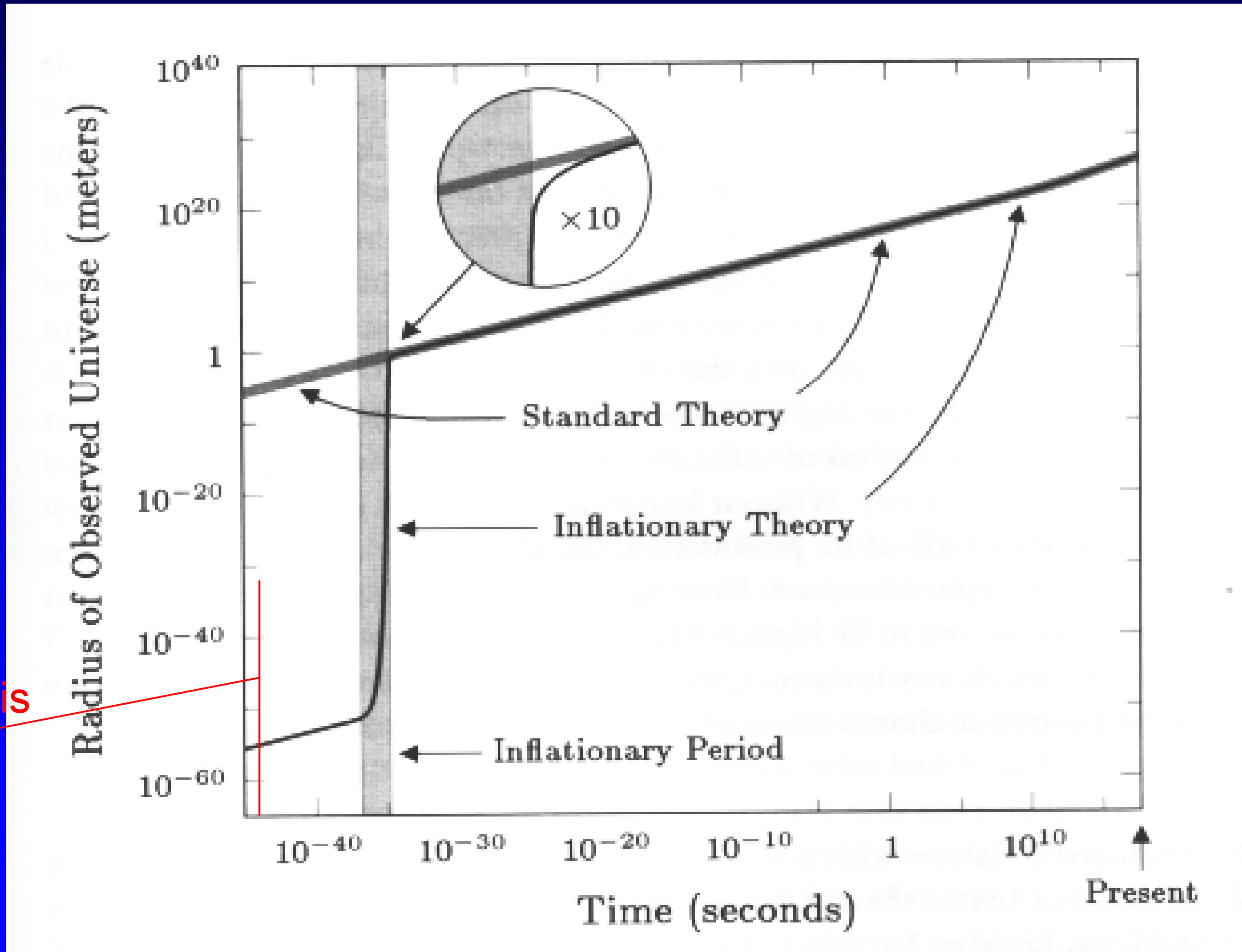
- For Ω to be between 0.1 and 2 now it must have been extremely close to 1 in the past

Inflation solution for:
Horizon Problem



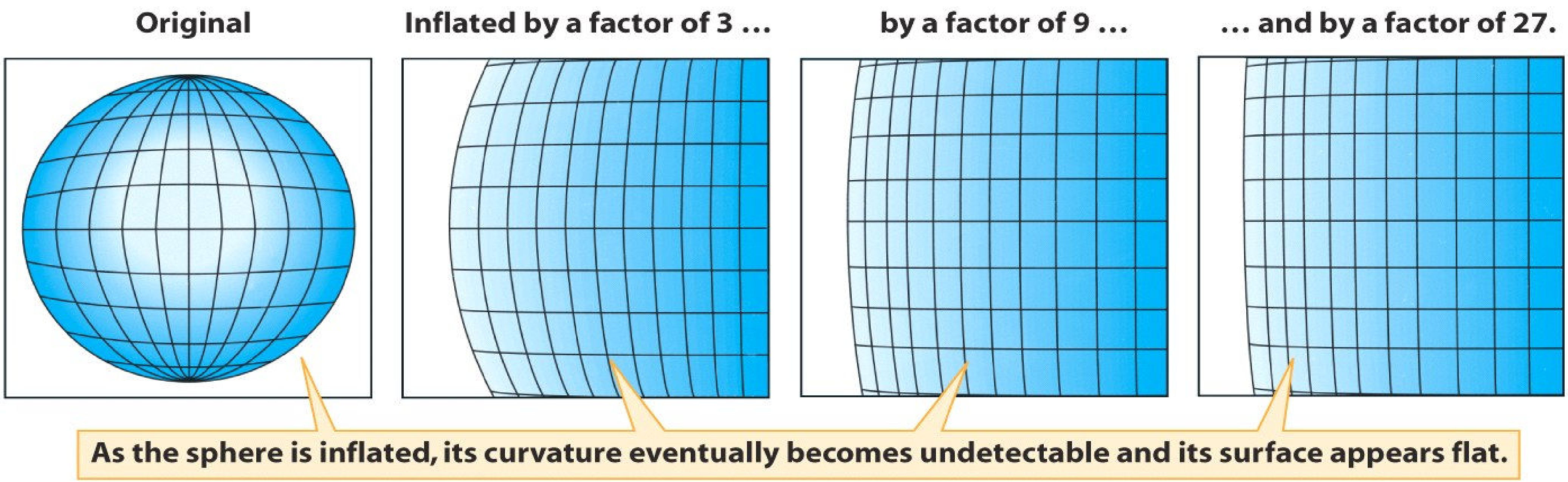
- If we incorporate an exponential expansion period into our calculation of the horizon at the time of last scattering, then the horizon becomes larger than the region we detect the cmb from

Inflation solution for:
Horizon Problem



light has
travelled this
distance

Inflation solution for:
Flatness problem



- A period of inflation drives Ω very, very close to 1 even if it was far from 1 before inflation

Scalar field

- The pressure and density of a scalar field Φ are

$$\rho = \frac{1}{2}\frac{1}{\hbar c^5}\dot{\phi}^2 + \frac{V(\phi)}{c^2}$$

and

$$P = \frac{1}{2}\frac{1}{\hbar c^3}\dot{\phi}^2 - V(\phi)$$

- If

$$\dot{\phi} \sim 0 \qquad \text{then} \qquad \frac{P}{c^2} = -\rho$$

- So inflation would occur if a scalar field (generically called inflaton) had a non-zero potential energy and only slowly changing
- Inflation is not happening now so the scalar field must be at its minimum

The inflaton rolls slowly down…

- The inflaton wants to roll down to its true vacuum, i.e. the energy minimum
- While you roll down you release energy by transforming potential energy into kinetic energy

Baryon Asymmetry

- Freeze-out calculation for nucleons:

'freeze-out' at $T \sim m_N/45$, with: $\frac{n_N}{n_\gamma} = \frac{n_{\bar{N}}}{n_\gamma} \sim 10^{-19}$

However the observed ratio is 10^9 times *bigger* for baryons, and *no* antibaryons are present, so there must have been an **initial asymmetry** of:

$$\frac{n_B - n_{\bar{B}}}{n_B + n_{\bar{B}}} \sim 10^{-9}$$

i.e. for every 10^9 baryon-antibaryon pairs there was *1 extra baryon*

Conditions for creating baryon asymmetry

1. Baryon number violation
2. *C* and *CP* violation
3. Departure from thermal equilibrium

↗ Why not at GUT transition?


COSMOLOGY MARCHES ON
Where did it all come from?
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S. Harris