(BEYOND) THE STANDARD MODEL

Wilfried Buchmüller
DESY

[see many books; European Schools of High-Energy Physics, CERN reports, e.g. WB, C. Lüdeling, hep-ph/0609174]

UniverseNet School, Oxford, September 2008

OUTLINE

- The fields of the Standard Model
- Why we believe in quantum field theory
- Divergencies and renormalisation
- Higgs sector and supersymmetry
- Unification and extra dimensions

What is the Standard Model?

The standard model of particle physics has the following key features:

- As a theory of elementary particles, it incorporates relativity and quantum mechanics, and therefore it is based on quantum field theory.

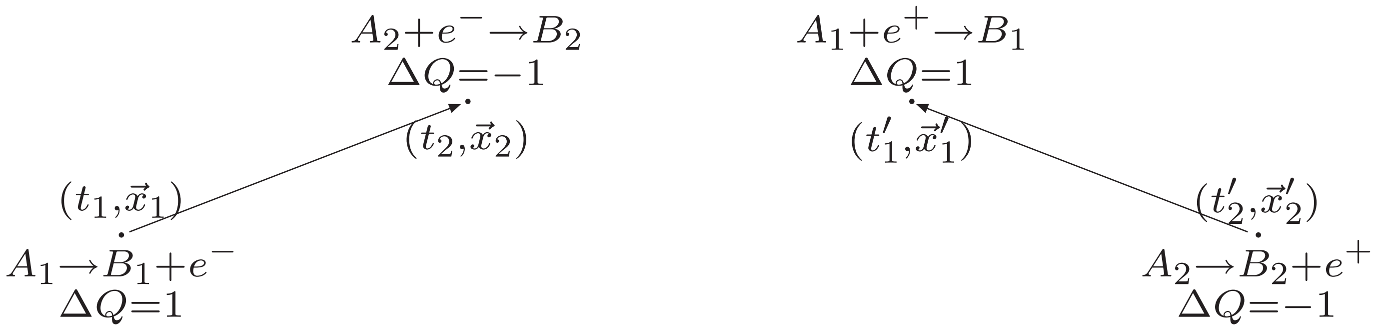
- Its predictive power rests on the regularisation of divergent quantum corrections and the renormalisation procedure which introduces scale–dependent "running couplings".

- Electromagnetic, weak, strong and also gravitational interactions are all related to local symmetries and described by Abelian and non-Abelian gauge theories.

- The masses of all particles are generated by two mechanisms: confinement and spontaneous symmetry breaking.

(1) The fields of the Standard Model

Special relativity and quantum mechanics lead to quantum field theory. Causality requires antiparticles (see Weinberg, in GR):



Consider two systems A_1 and A_2 at $\vec{x}_1$ and $\vec{x}_2$; at t_1, A_1 emits electron and turns into B_1; at $t_2 > t_1$, electron is absorbed by A_2 which turns into B_2.

Watch system from moving frame with relative velocity $\vec{v}$; emission still before absorption (causality)? In boosted frame,

$$t'_2 - t'_1 = \gamma\,(t_2 - t_1) + \gamma\vec{v}\,(\vec{x}_2 - \vec{x}_1)\ , \quad \gamma = \frac{1}{\sqrt{1 - \vec{v}^{\,2}}}\,;$$

$t'_2 - t'_1$ only negative for spacelike distances, i.e. $(t_2 - t_1)^2 - (\vec{x}_1 - \vec{x}_2)^2 < 0$, not possible in special relativity; within classical physics, causality is OK.

In quantum mechanics, uncertainty relation leads to "fuzzy" light cone, non-zero propagation probability of electron for slightly spacelike distances,

$$(t_2 - t_1)^2 - (\vec{x}_1 - \vec{x}_2)^2 \gtrsim -\frac{\hbar^2}{m^2}\,.$$

Causality is saved by introducing antiparticles. In moving frame, emission of positron at t'_2, followed by absorption at $t'_1 > t'_2$.

In relativistic theory, particles cannot be localized below their Compton wavelength,

$$\Delta x \geq \frac{\hbar}{mc}\,.$$

For shorter distances, momentum uncertainty $\Delta p > mc$ implies contributions from multiparticle states. Corresponding Fock space,

vacuum:	$\|0\rangle \ , \quad a(k)\|0\rangle = b(k)\|0\rangle = 0$
one-particle states:	$a^\dagger(k)\|0\rangle \ , \ b^\dagger(k)\|0\rangle$
two-particle states:	$a^\dagger(k_1)a^\dagger(k_2)\|0\rangle \ , \ a^\dagger(k_1)b^\dagger(k_2)\|0\rangle \ , \ b^\dagger(k_1)b^\dagger(k_2)\|0\rangle$
	$\vdots$

Dynamics conveniently described by means of field operators,

$$\phi(x) = \int \overline{\mathrm{d}k} \left(e^{-\mathrm{i}kx} a(k) + e^{\mathrm{i}kx} b^\dagger(k)\right) .$$

$\rightarrow$ Lagrange formalism, canonical quantisation, path integral methods,...

The Standard Model: a chiral gauge theory

The SM is theory of fields with spins 0, $\frac{1}{2}$ and 1. The fermions (matter fields) can be viewed as big vector containing left-handed spinors only,

$$\Psi_L^T = \Big(\underbrace{q_{L1}, u^c_{R1}, e^c_{R1}, d^c_{R1}, l_{L1}, (n^c_{R1})}_{\text{1st family}}, \underbrace{q_{L2}, \ldots}_{\text{2nd}}, \underbrace{\ldots, (n^c_{R3})}_{\text{3rd}} \Big) ,$$

with quarks and leptons, in threefold family replication; quarks are triplets of colour (index $\alpha = 1, 2, 3$); left-handed quarks and leptons are doublets of weak isospin,

$$q^\alpha_{Li} = \begin{pmatrix} u^\alpha_{Li} \\ d^\alpha_{Li} \end{pmatrix} \qquad l_{Li} = \begin{pmatrix} \nu_{Li} \\ e_{Li} \end{pmatrix} ,$$

with family index $i = 1, 2, 3$; evidence for right-handed neutrino n_R because of neutrino masses deduced from neutrino oscillation experiments (?)

L and R denote left- and right-handed fields, eigenstates of the chiral projection operators P_L or P_R; c indicates charge conjugate field (antiparticle); note: charge conjugate of right-handed field is left-handed,

$$P_L\psi_L \equiv \frac{1-\gamma^5}{2}\psi_L = \psi_L\ , \quad P_L\psi_R^c = \psi_R^c\ , \quad P_L\psi_R = P_L\psi_L^c = 0\ ,$$
$$P_R\psi_R \equiv \frac{1+\gamma^5}{2}\psi_R = \psi_R\ , \quad P_R\psi_L^c = \psi_L^c\ , \quad P_R\psi_L = P_R\psi_R^c = 0\ .$$

All fields in big column vector of fermions are chosen left-handed, altogether 48 chiral fermions! Since left- and right-handed fermions carry different weak isospin, the SM is a chiral gauge theory. Threefold replication of quark-lepton families: puzzle to be explained by physics beyond the SM.

The spin-1 particles are the gauge bosons whose exchange yields the fundamental interactions in the SM,

$$G_\mu^A,\ A = 1,\ldots,8: \qquad \text{gluons of strong interactions}$$
$$W_\mu^I,\ I = 1,2,3;\ B_\mu: \ W \text{ and } B \text{ bosons of electroweak interactions},$$

associated with the local symmetry group

$$G_{\mathsf{SM}} = \mathsf{SU}(3)_C \times \mathsf{SU}(2)_W \times \mathsf{U}(1)_Y \ ,$$

where C, W, and Y denote colour, weak isospin and hypercharge.

Coupling of vector fields (big matrix A_μ, includes generators of gauge group) to fermions via covariant derivative D_μ (cf. GR),

$$D_\mu \Psi_L = \left(\partial_\mu \mathbb{1} + g A_\mu\right) \Psi_L \ ;$$

self-coupling of gauge bosons from field strength,

$$F_{\mu\nu} = -\frac{\mathsf{i}}{g}\left[D_\mu, D_\nu\right] \, .$$

Final, crucial ingredient of SM is the Higgs field Φ, only spin-0 field in the theory, doublet of weak isospin. It couples left- and right-handed fermions together and generates all mass terms! Full SM Lagrangean has rather simple structure

$$\mathcal{L} = -\frac{1}{2}\,\mathrm{tr}\left[F_{\mu\nu}F^{\mu\nu}\right] + \overline{\Psi}_L \mathsf{i}\gamma^\mu D_\mu \Psi_L + \mathrm{tr}\left[\left(D_\mu\Phi\right)^\dagger D^\mu\Phi\right]$$
$$+\mu^2\,\Phi^\dagger\Phi - \frac{1}{2}\lambda\left(\Phi^\dagger\Phi\right)^2 + \left(\frac{1}{2}\Psi_L^T C h \Phi \Psi_L + \text{h.c.}\right) \, ,$$

with matrix h of Yukawa couplings. All couplings are dimensionless, all masses are generated via the Higgs mechanism, which gives vacuum expectation value to Higgs field,

$$\langle \Phi \rangle \equiv v = 174\,\text{GeV}\;;$$

Higgs boson likely to be discovered at the LHC.

Phenomenology

SM Lagrangean describes successfully all areas of particle physics:

- SU(3) subgroup corresponds to QCD, theory of strong interactions; most important phenomena: asymptotic freedom and confinement; quarks and gluons appear as free particles at short distances, probed in deep-inelastic scattering, but are confined into mesons and baryons at large distances.

- SU(2) $\times$ U(1) subgroup describes electroweak sector of SM; broken to the $\mathrm{U}(1)_{\mathrm{em}}$ of QED by the Higgs mechanism, leading to massive W and Z bosons responsible for charged and neutral current weak interactions.

- Yukawa interaction term includes different pieces for quarks and leptons:

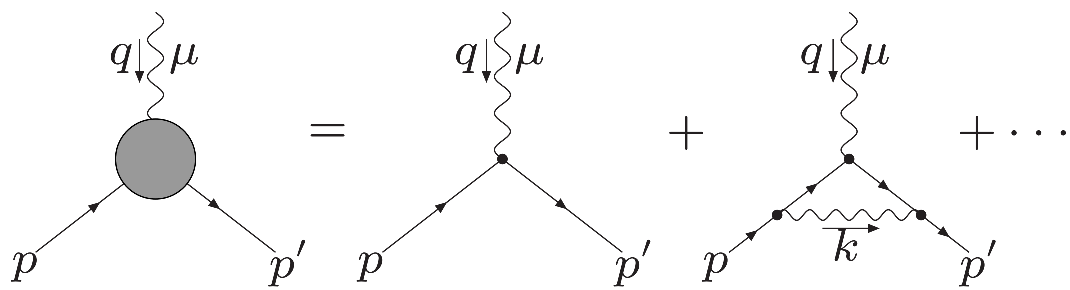
$$\begin{aligned}\frac{1}{2}\Psi_L^T Ch\Phi\Psi_L &= h_{u\,ij}\bar{u}_{Ri}q_{Lj}\Phi + h_{d\,ij}\bar{d}_{Ri}q_{Lj}\widetilde{\Phi} \\ &\quad + h_{e\,ij}\bar{e}_{Ri}l_{Lj}\widetilde{\Phi} + h_{n\,ij}\bar{n}_{Ri}l_{Lj}\Phi\ ,\end{aligned}$$

with family indices $i, j = 1, 2, 3$, and $\widetilde{\Phi}_a = \epsilon_{ab}\Phi_b^*$. Higgs vacuum expectation value $\langle\Phi\rangle = v$ generates mass terms; 'misalignement' of up-type- and down-type-quarks leads to CKM matrix and flavour physics; last two terms yield lepton masses and neutrino mixings.

(2) Why we believe in quantum field theory

Perturbative expansion is most impressive !! Also non-perturbative methods [lattice gauge theory] very successful. Do interacting quantum field theories in four dimensions exist ??

Classical example: anomalous magnetic moment of the electron (Schwinger 1948)



The electromagnetic current is decomposed via the Gordon identity into convection and spin currents,

$$\bar{u}(p')\gamma^\mu u(p) = \bar{u}(p')\left(\frac{(p+p')^\mu}{2m} + \frac{\mathsf{i}}{2m}\sigma^{\mu\nu}\left(p'-p\right)_\nu\right)u(p)\,.$$

First term: flow of charged particles, same as for scalar particles; second term: spin current, relevant for magnetic moment; Landé factor of electron is $g_e = 2$. One-loop vertex correction,

$$\begin{aligned} \mathsf{i}e\Gamma^\mu(p,q) &= (-\mathsf{i}e)^3 \int \frac{\mathsf{d}^4k}{(2\pi)^4}\frac{-\mathsf{i}g_{\rho\sigma}}{k^2+\mathsf{i}\varepsilon}\gamma^\rho \frac{\mathsf{i}\left(\not{p}'-\not{k}+m\right)}{(p'-k)^2-m^2+\mathsf{i}\varepsilon}\gamma^\mu \\ &\qquad \times \frac{\mathsf{i}\left(\not{p}-\not{k}+m\right)}{(p-k)^2-m^2+\mathsf{i}\varepsilon}\gamma^\sigma\,. \end{aligned}$$

After some manipulations, finite result, expressed in terms of fine structure constant $\alpha = e^2/(4\pi)$,

$$\mathrm{i}e\bar{u}(p')\Gamma^\mu u(p) = +\mathrm{i}e\bar{u}(p')\left(\frac{\alpha}{2\pi}\frac{\mathrm{i}}{2m}\sigma^{\mu\nu}q_\nu + \cdots\right)u(p)\,;$$

dots represent contributions not $\propto \sigma^{\mu\nu}q_\nu$. Beyond one loop, divergencies and renormalisation required.

Comparison with Gordon decomposition gives one-loop correction to Landé factor,

$$g_e = 2\left(1 + \frac{\alpha}{2\pi}\right)\,,$$

i.e. anomalous magnetic moment $a_e = (g_e - 2)/2$.

Today: three loops known analytically, four loops numerically (success story over 50 years); agreement between theory and experiment most impressive:

$$
\begin{aligned}
a_e^{\text{exp}} &= (1159652185.9 \pm 3.8) \cdot 10^{-12} \, , \\
a_e^{\text{th}} &= (1159652175.9 \pm 8.5) \cdot 10^{-12} \, ,
\end{aligned}
$$

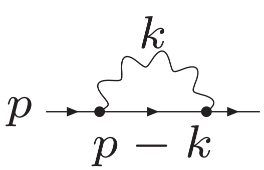
cornerstone of quantum field theory [Note: QED is inconsistent theory !]

Further tests of QFTH: more high-order calculations in QED; electroweak theory: non-Abelian gauge theory, precision analysis of LEP data, expectations for LHC, in particular Higgs boson mass; QCD: higher-order calculations of DIS, heavy quark physics, jets, parton evolution etc (but less clean).

(3) Divergencies and renormalisation

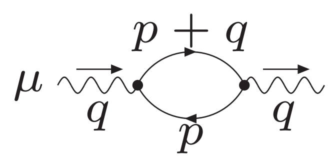
In the perturbative expansion ultraviolet divergencies occur, which require regularisation and renormalisation; typical one-loop integral, evaluated in $d = 4 - \epsilon$ dimensions (regularisation):

$$\mu^{\epsilon} \int \frac{\mathrm{d}^4 k_{\mathsf{E}}}{(2\pi)^4} \frac{1}{\left(k_{\mathsf{E}}^2 + C\right)^2} = \frac{\mu^{\epsilon}\Gamma\left(2 - \frac{d}{2}\right)}{(4\pi)^{d/2}\,\Gamma(2)} \frac{1}{C^{2-d/2}} = \frac{1}{8\pi^2}\frac{1}{\epsilon} + \cdots$$



$= -\mathrm{i}\Sigma\,(p)$

(a) Electron self-energy



$= -\mathrm{i}\Pi_{\mu\nu}\,(q)$

(b) Vacuum polarisation

Example: vacuum polarisation, second rank tensor; requirement of gauge invariance,

$$q^{\mu}\Pi_{\mu\nu}(q) = 0 \; ,$$

together with Lorentz invariance,

$$\Pi_{\mu\nu}(q) \quad = \left(g_{\mu\nu}q^2 - q_{\mu}q_{\nu}\right)\Pi\left(q^2\right) \; ,$$

yields scalar quantity $\Pi(q^2)$ which has divergent part,

$$\Pi\left(q^2\right) = \frac{2\alpha}{3\pi}\frac{1}{\epsilon} + \mathcal{O}(1) \, .$$

Renormalisation: divergencies can be absorbed into "bare" fields and "bare" parameters; they are not observable. Explicitly, for QED:

$$\mathcal{L} = -\frac{1}{4}\left(\partial_\mu A_{0\,\nu} - \partial_\nu A_{0\,\mu}\right)\left(\partial^\mu A^{0\,\nu} - \partial^\nu A^{0\,\mu}\right)$$
$$+\bar{\psi}_0\left(\gamma^\mu\left(\mathrm{i}\partial_\mu - e_0 A_{0\,\mu}\right) - m_0\right)\psi_0\,.$$

"Renormalised fields" A_μ and ψ and "renormalised parameters" e and m are obtained from bare ones by multiplicative rescaling,

$$A_{0\,\mu} = \sqrt{Z_3}A_\mu\,, \quad e_0 = \frac{Z_1}{Z_2\sqrt{Z_3}}\mu^{2-d/2}e\ \ldots$$

Note: coupling, electron mass and fields now depend on mass parameter μ, $e = e(\mu)$,....

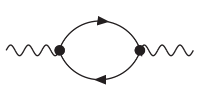
QED Lagrangean in terms of renormalized fields and parameters,

$$\mathcal{L} = -\frac{1}{4}\left(\partial_\mu A_\nu - \partial_\nu A_\mu\right)\left(\partial^\mu A^\nu - \partial^\nu A^\mu\right) + \bar{\psi}\left(\gamma^\mu\left(\mathrm{i}\partial_\mu - eA_\mu\right) - m\right)\psi + \Delta\mathscr{L} \ ,$$

where $\Delta\mathcal{L}$ contains the divergent counterterms,

$$\Delta\mathcal{L} = -\left(Z_3 - 1\right)\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \left(Z_2 - 1\right)\bar{\psi}\mathrm{i}\not{\partial}\psi$$
$$-\left(Z_m - 1\right)m\bar{\psi}\psi - \left(Z_1 - 1\right)e\bar{\psi}\not{A}\psi \ .$$

Vacuum polarisation now has two contributions to $\mathcal{O}\left(\alpha\right)$,

 $+$  $= -\mathrm{i}\left(g_{\mu\nu}q^2 - q_\mu q_\nu\right)\left(\frac{2\alpha}{3\pi}\frac{1}{\epsilon} + \left(Z_3 - 1\right) + \mathcal{O}\left(1\right)\right) \ ,$

Result finite for the choice

$$Z_3 = 1 - \frac{2\alpha}{3\pi}\frac{1}{\epsilon} + \mathcal{O}(1) \ .$$

After absorbtion of divergences into renormalised parameters and fields, one can take limit $\epsilon \to 0$. The theory yields well-defined relations between physical observables. Divergencies can be removed to all orders in loop expansion for renormalisable theories! QED and the standard model belong to this class. Proof is highly non-trivial, major achievement in quantum field theory!

Running couplings in QED and QCD

Contrary to bare coupling e_0, renormalised coupling $e(\mu)$ depends on renormalisation scale μ (use Ward identity $Z_1 = Z_2$),

$$e_0 = \frac{Z_1}{Z_2\sqrt{Z_3}}\mu^{-2+d/2}e(\mu) = e(\mu)\mu^{-\epsilon/2}Z_3^{-\frac{1}{2}} \,,$$

Remarkably, scale dependence is determined by divergencies! Expand in ϵ and $e(\mu)$ (use $\alpha = e^2/(4\pi)$),

$$\begin{aligned} e_0 &= e(\mu)\left(1 - \frac{\epsilon}{2}\ln\mu + \cdots\right)\left(1 + \frac{1}{\epsilon}\frac{\alpha}{3\pi} + \cdots\right) \\ &= e(\mu)\left(\frac{1}{\epsilon}\frac{e^2(\mu)}{12\pi^2} + 1 - \frac{e^2(\mu)}{24\pi^2}\ln\mu + \mathcal{O}\left(\epsilon, e^4(\mu)\right)\right) \;; \end{aligned}$$

differentiation with respect to μ,

$$0 = \mu \frac{\partial}{\partial \mu} e_0 = \mu \frac{\partial}{\partial \mu} e - \frac{e^3}{24\pi^2} + \mathcal{O}\left(e^5\right) \ ,$$

gives renormalisation group equation,

$$\mu \frac{\partial}{\partial \mu} e = \frac{e^3}{24\pi^2} + \mathcal{O}\left(e^5\right) \equiv \beta(e) \ ,$$

with β-function

$$\beta(e) = \frac{b_0}{(4\pi)^2} e^3 + \mathcal{O}\left(e^5\right) \ , \quad b_0 = \frac{2}{3} \ .$$

Integration yields running coupling in terms of coupling at reference scale μ_1,

$$\alpha(\mu)=\frac{\alpha\left(\mu_1\right)}{1-\alpha\left(\mu_1\right)\frac{b_0}{(2\pi)}\ln\frac{\mu}{\mu_1}}\;;$$

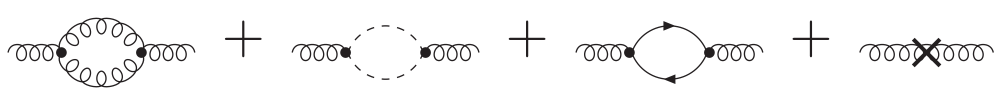
since $b_0>0$, coupling increases with μ until it approaches the Landau pole where perturbation theory breaks down!

What is the meaning of a scale dependent coupling? For physical quantities, e.g. scattering amplitude at momentum transfer q^2, perturbative expansion generates terms $\propto e^2(\mu)\log\left(q^2/\mu^2\right)$. Hence, expansion unreliable unless one chooses $\mu^2\sim q^2$. Running coupling $e^2\left(q^2\right)$ therefore represents effective interaction strength at momentum (or energy) scale q^2 or, correspondingly, at distance $r\sim 1/q$.

In QED, because of positive β function, effective coupling strength decreases at large distances; effect of "vacuum polarisation": electron-positron pairs from the vacuum screen bare charge at distances larger than electron Compton wavelength. In Thompson limit, $\alpha(0) = \frac{1}{137}$, increases to $\alpha\big(M_Z^2\big) = \frac{1}{127}$ [important input in electroweak precision tests, hints for "new physics"].

Running Coupling in QCD

Contributions to running coupling in non-Abelian gauge theories, in particular QCD:



$\rightsquigarrow Z_3$.

Renormalised coupling can be defined as in QED,

$$g_0 = \frac{Z_1}{Z_2\sqrt{Z_3}}\mu^{-2+d/2}\, g\,.$$

Scale dependence from coefficients of $1/\epsilon$-divergences, depend on number of colours ($N_c^2 - 1$) and flavous (N_f),

$$\mu\frac{\partial}{\partial\mu}g = \frac{b_0}{(4\pi)^2}g^3 + \mathcal{O}\left(g^5\right)\,, \quad b_0 = -\left(\frac{11}{3}N_{\mathsf{c}} - \frac{4}{3}N_{\mathsf{f}}\right)\,.$$

Coefficient negative for $N_f < 11N_{\mathsf{c}}/4$, i.e. QCD !! Coupling then decreases at high momentum transfers or short distances: asymptotic freedom. As a consequence, in deep-inelastic scattering quarks inside proton quasi-free particles $\rightarrow$ parton model, basis for treatment of collisions at the LHC!

Coupling scale μ can be expressed in terms of coupling at reference scale

μ_1, e.g. $\mu_1 = m_Z$,

$$\alpha(\mu) = \frac{\alpha\left(\mu_1\right)}{1 + \alpha\left(\mu_1\right) \frac{|b_0|}{(2\pi)} \ln \frac{\mu}{\mu_1}} .$$

Analogue of Landau pole now at small μ, i.e. large distances. QCD with $N_\mathsf{c} = 3$ and $N_\mathsf{f} = 6$: pole at "QCD scale" $\mu \sim \Lambda_\mathsf{QCD} \simeq 300$ MeV. Gluons and quarks then strongly coupled and colour confined. Size and masses of hadrons,

$$r_\mathsf{had} \sim \Lambda_\mathsf{QCD}^{-1} \sim 0.7 \text{ fm} , \quad m_\mathsf{proton} \sim 3\ \Lambda_\mathsf{QCD} \sim 1 \text{ GeV} .$$

Origin of mass of ordinary matter mostly non-perturbative !!

(4) Higgs sector and supersymmetry

All masses in the SM are generated by Higgs mechanism, based on effective potential which allows "spontaneous symmetry breaking",

$$\begin{aligned} \mathcal{L} &= (D_\mu \Phi)^\dagger (D^\mu \Phi) - V\left(\Phi^\dagger \Phi\right) , \\ D_\mu \Phi &= \left(\partial_\mu + \mathrm{i} g W_\mu - \frac{\mathrm{i}}{2} g' B_\mu\right) \Phi , \\ V\left(\Phi^\dagger \Phi\right) &= -\mu^2 \, \Phi^\dagger \Phi + \frac{1}{2} \lambda \left(\Phi^\dagger \Phi\right)^2 , \quad \mu^2 > 0 ; \end{aligned}$$

potential has minimum away from origin, at $\Phi^\dagger \Phi = v^2 \equiv \mu^2/\lambda$, which defines the vacuum. In unitary gauge,

$$\Phi = \begin{pmatrix} 0 \\ v + \frac{1}{\sqrt{2}} H(x) \end{pmatrix} , \quad H = H^* .$$

The Higgs Lagrangean generates all mass terms,

$$
\begin{aligned}
\mathcal{L} \;=\; & \frac{\lambda}{2} v^4 \\
& + \frac{1}{2}\partial_\mu H\, \partial^\mu H - \lambda v^2 H^2 + \frac{\lambda}{2} v H^3 + \frac{\lambda}{8} H^4 \\
& + \frac{1}{4}\left(v + \frac{1}{\sqrt{2}} H\right)^2 \left(W^1_\mu, W^2_\mu, W^3_\mu, B_\mu\right) \begin{pmatrix} g^2 & 0 & & \\ 0 & g^2 & & \\ & & g^2 & gg' \\ & & gg' & g'^2 \end{pmatrix} \begin{pmatrix} W^{1\,\mu} \\ W^{2\,\mu} \\ W^{3\,\mu} \\ B^\mu \end{pmatrix} ;
\end{aligned}
$$

first term: vacuum energy density (?), then Higgs and vector boson masses:

- $W^\pm$: $M_W^2 = \frac{1}{2} g^2 v^2$; Z: $M_Z^2 = \frac{1}{2}\left(g^2 + g'^2\right) v^2$; γ: $M_\gamma = 0$,
- Higgs: $m_H^2 = 2\lambda v^2$.

Higgs mass bounds

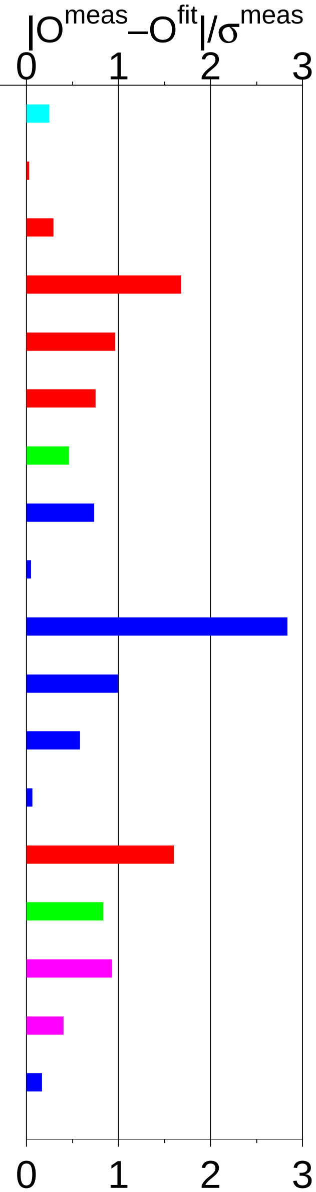
Most important quantity, for LHC and extrapolations beyond: Higgs mass! Current experimental bounds:

- Higgs not seen at LEP: $m_H > 114$ GeV.
- Higgs contributes to radiative corrections, ρ-parameter etc Global fit to precision measurements (see Figure) summarised in blue-band plot (small plots:1997, 2001, 2003, 2005; big plot 2006); present 95% confidence level upper bound:

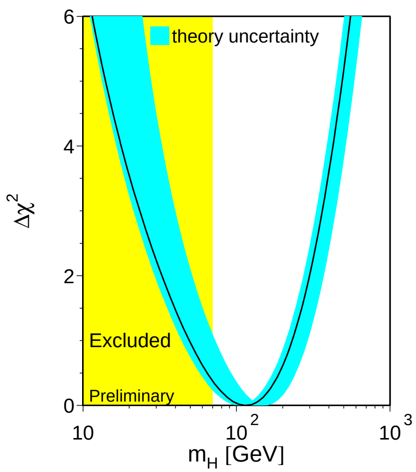
$$m_H < 185 \text{ GeV}\,.$$

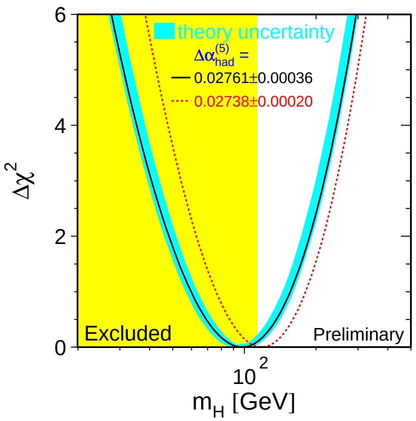
Note: loop corrections strongly dependent on top mass (main reason for variations in early years).

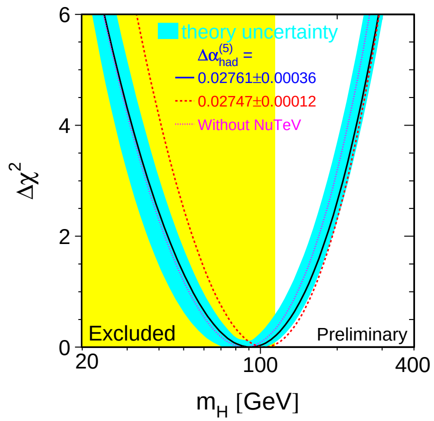
	Measurement	Fit
$\Delta\alpha^{(5)}_{had}(m_Z)$	0.02758 ± 0.00035	0.02767
m_Z [GeV]	91.1875 ± 0.0021	91.1874
Γ_Z [GeV]	2.4952 ± 0.0023	2.4959
σ^0_{had} [nb]	41.540 ± 0.037	41.478
R_l	20.767 ± 0.025	20.743
$A^{0,l}_{fb}$	0.01714 ± 0.00095	0.01643
$A_l(P_\tau)$	0.1465 ± 0.0032	0.1480
R_b	0.21629 ± 0.00066	0.21581
R_c	0.1721 ± 0.0030	0.1722
$A^{0,b}_{fb}$	0.0992 ± 0.0016	0.1037
$A^{0,c}_{fb}$	0.0707 ± 0.0035	0.0742
A_b	0.923 ± 0.020	0.935
A_c	0.670 ± 0.027	0.668
A_l(SLD)	0.1513 ± 0.0021	0.1480
$\sin^2\theta^{lept}_{eff}(Q_{fb})$	0.2324 ± 0.0012	0.2314
m_W [GeV]	80.404 ± 0.030	80.376
Γ_W [GeV]	2.115 ± 0.058	2.092
m_t [GeV]	172.5 ± 2.3	172.9

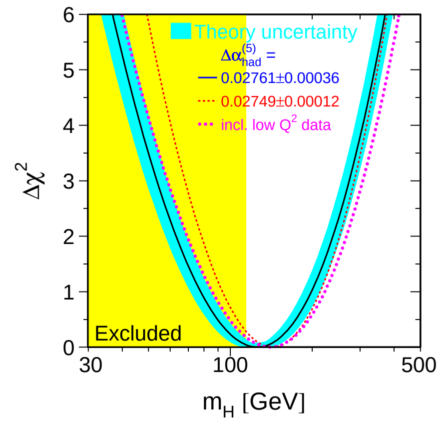


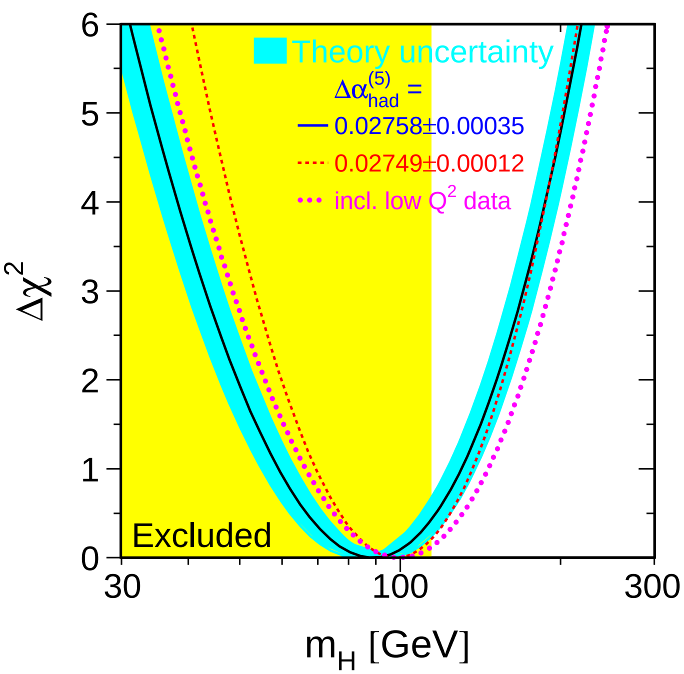
Global fit to electroweak precision data.


theory uncertainty
Excluded
Preliminary
Δχ²
m_H [GeV]
10
10²
10³


theory uncertainty
Δα_had^(5) =
0.02761±0.00036
0.02738±0.00020
Excluded
Preliminary
Δχ²
m_H [GeV]


theory uncertainty
Δα_had^(5) =
0.02761±0.00036
0.02747±0.00012
Without NuTeV
Excluded
Preliminary
Δχ²
m_H [GeV]


Theory uncertainty
Δα_had^(5) =
0.02761±0.00036
0.02749±0.00012
incl. low Q² data
Excluded
Δχ²
m_H [GeV]


Theory uncertainty
Δα_had^(5) =
0.02758±0.00035
0.02749±0.00012
incl. low Q² data
Excluded
Δχ²
m_H [GeV]

Theoretical bounds on the Higgs mass arise in SM from two consistency requirements: (non-)triviality and vacuum stability; in minimal supersymmetric standard model (MSSM) the Higgs self-coupling is given by gauge couplings, which yields the upper bound $m_H \lesssim 135$ GeV.

Theoretical mass bounds follow from scale dependence of couplings; most relevant: quartic Higgs self-coupling λ and top quark Yukawa coupling $h_t = m_t/v$; coupled system of renormalisation group equations:

$$\begin{aligned}
\mu\frac{\partial}{\partial\mu}\lambda(\mu) &= \frac{1}{(4\pi)^2}\left(12\lambda^2 - 12h_t^4 + \ldots\right) = \beta_\lambda(\lambda, h_t)\ , \\
\mu\frac{\partial}{\partial\mu}h_t(\mu) &= \frac{h_t}{(4\pi)^2}\left(\frac{9}{2}h_t^2 - 8g_s^2 + \ldots\right) = \beta_\lambda(\lambda, h_t)\ ;
\end{aligned}$$

h_t decreases with increasing μ, behaviour of $\lambda(\mu)$ depends on initial value $\lambda(v)$, i.e., on the Higgs mass.

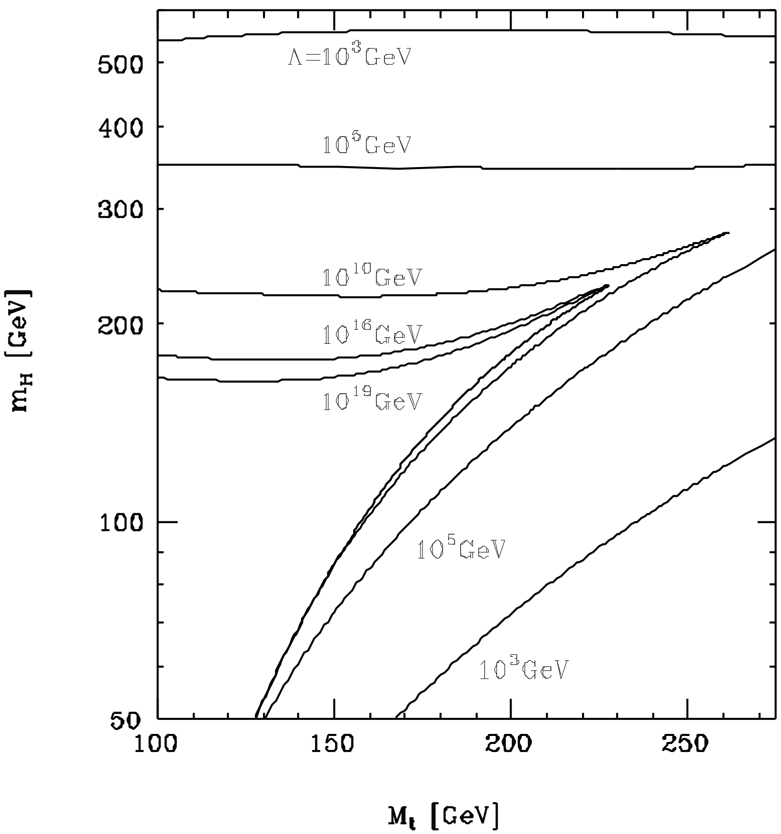
Consistency of SM from electroweak scale v up to some high-energy cutoff Λ, yields conditions for running couplings in range $v < \mu < \Lambda$:

- Triviality bound: $\lambda(\mu) < \infty$; if λ would hit Landau pole at some scale $\mu_\mathsf{L} < \Lambda$, finite value $\lambda(\mu_\mathsf{L})$ would require $\lambda(v) = 0$, i.e., theory would be "trivial".

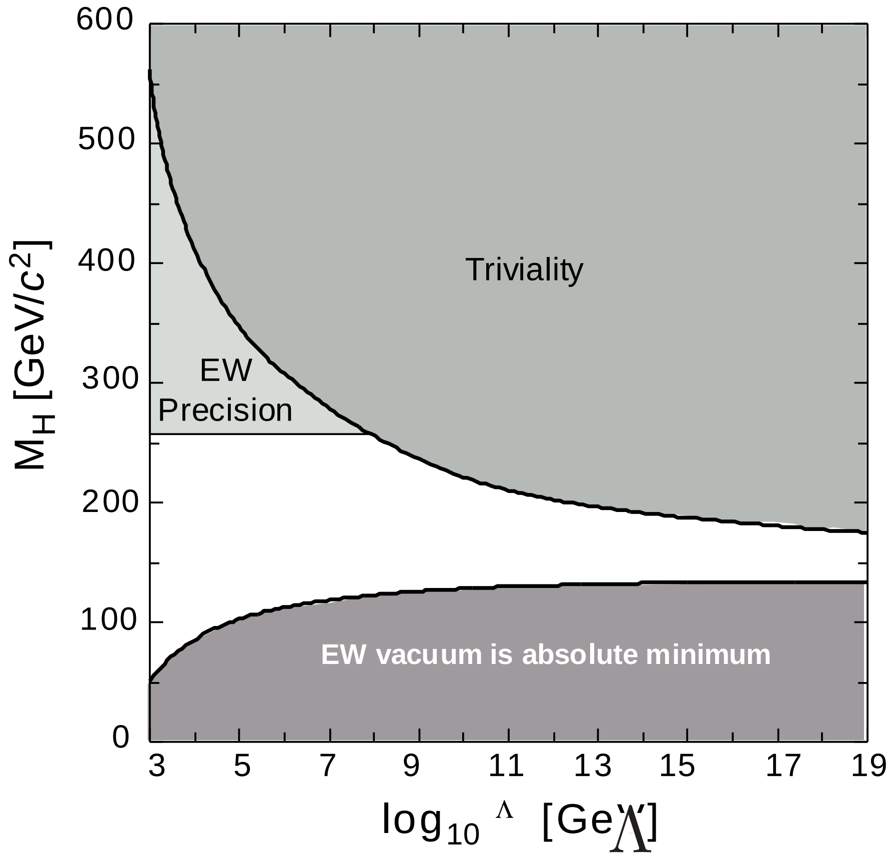
- Vacuum stability bound: $\lambda(\mu) > 0$; if λ would become negative, Higgs potential would be unbounded from below anymore $\rightarrow$ electroweak vacuum no longer ground state!

These requirements define allowed regions in the m_H-m_t–plane as function of cutoff Λ; Higgs mass range for known top mass and $\Lambda \sim \Lambda_\mathsf{GUT} \sim 10^{16}$ GeV,

$$130 \text{ GeV} < m_H < 180 \text{ GeV} .$$



(c)



(d)

How far should we extrapolate beyond the electroweak scale?

Attractive extension of SM: SUPERSYMMETRY, in particular the 'minimal' supersymmetric SM (MSSM); number of fields are doubled:

$$\mathrm{SM}\,,\ \{\Phi_i\}\ \rightarrow\ \mathrm{MSSM},\ \{\Phi_{ia}\}\ ,$$

where $i = q, l, W, g, \gamma, \ldots$ and

$$\begin{aligned} a = 1: &\qquad \text{old particles}\ ,\ p_i\ , \\ a = 2: &\qquad \text{new (s)particles}\ ,\ \tilde{p}_i\ ; \end{aligned}$$

improves ultraviolet behaviour, attractive theoretical structure; MSSM has two Higgs doublets; rich phenomenology at LHC (already in RPP since many years, conferences SUSY0x,...); phenomenological problems at low energies: proton decay, flavour changing neutral curents, dipole moments, gravitino problem in cosmology...(?)

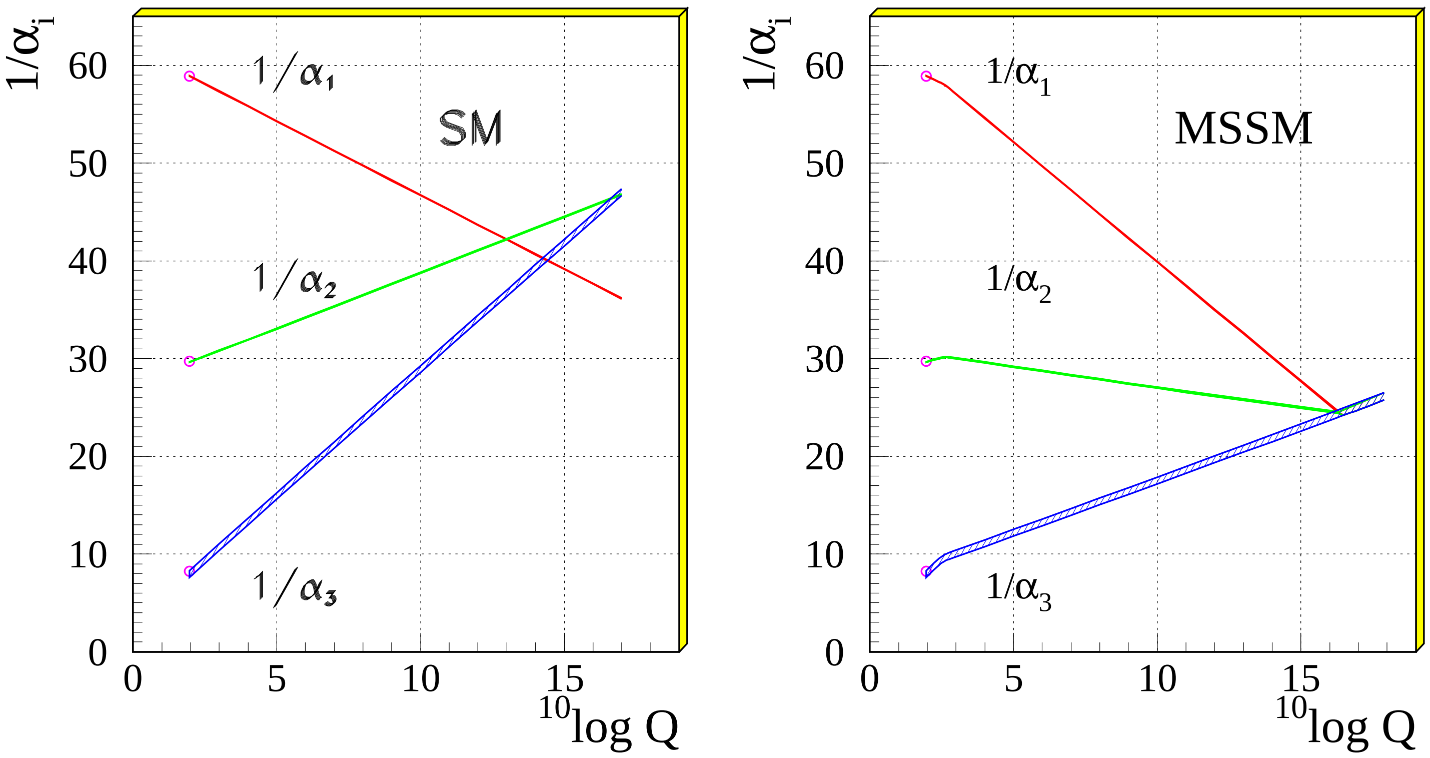
Supersymmetry stabilizes Higgs vacuum expectation value v w.r.t. radiative corrections, hence in MSSM supersymmetry breaking related to electroweak mass scale,

$$\Delta m^2_{susy} = m^2_{\tilde{p}} - m^2_p \sim v^2 \; .$$

Predictions: spectrum of (s)particles with masses in the range 100 GeV - 2 TeV; could be discovered and studied at LHC; mass spectrum depends on mechanism of supersymmetry breaking.

also important: precision measurements of low energy processes, in particular $\mu \to e\gamma$: $BR > 10^{-14}$ for large class of models, could be discovered in near future at PSI; attractive dark matter candidates; unification of gauge couplings,...(already SM impressive)...

Unification of the Coupling Constants in the SM and the minimal MSSM



(5) Unification and Higher Dimensions

Grand unified theories (GUTs) are natural extension of the standard model; quarks and leptons form SU(5) multiplets (Georgi,Glashow),

$$\mathbf{10} = (q_L, u_R^c, e_R^c) \ , \quad \mathbf{5}^* = (d_R^c, l_L) \ , \quad (\mathbf{1} = \nu_R) \ ,$$

or SU(4)×SU(2)×SU(2) multiplets (Pati,Salam),

$$(\mathbf{4}, \mathbf{2}, \mathbf{1}) = (q_L, l_L) \ , \quad (\mathbf{4}^*, \mathbf{1}, \mathbf{2}) = (u_R^c, d_R^c, \nu_R^c, e_R^c) \ ;$$

all quarks and leptons of one generation are unified in a single multiplet in the GUT group SO(10),

$$\mathbf{16} = \mathbf{10} + \mathbf{5}^* + \mathbf{1} = (\mathbf{4}, \mathbf{2}, \mathbf{1}) + (\mathbf{4}^*, \mathbf{1}, \mathbf{2}) \ .$$

Important hint for unification, in addition to gauge coupling unification: small neutrino masses; simple explanation by seesaw mechanism via mixing ($m_D = h_\nu v$) with heavy right-handed neutrinos (M),

$$m_\nu = -m_D \frac{1}{M} m_D^T \ .$$

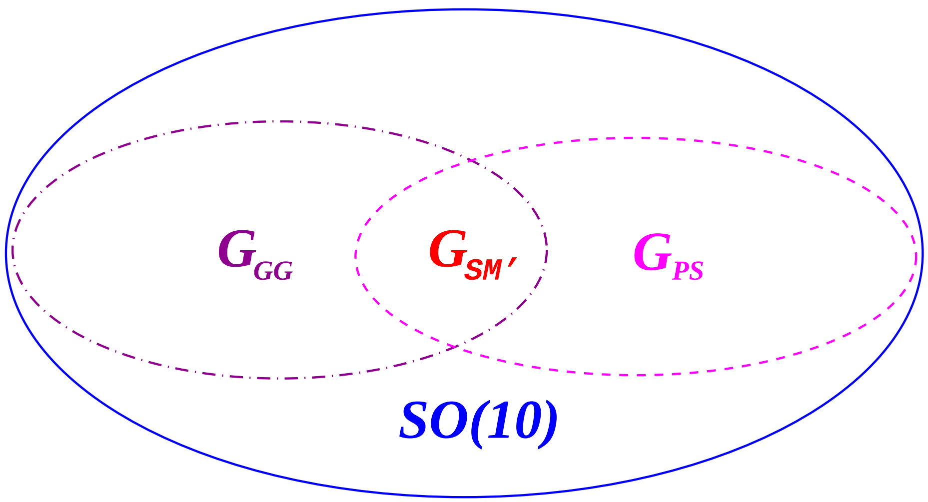
Estimate of largest light-neutrino mass, with $M \sim \Lambda_{GUT} \sim 10^{15}$ GeV,

$$m_3 \sim \frac{v^2}{M} \sim 0.01 \text{ eV} \ ,$$

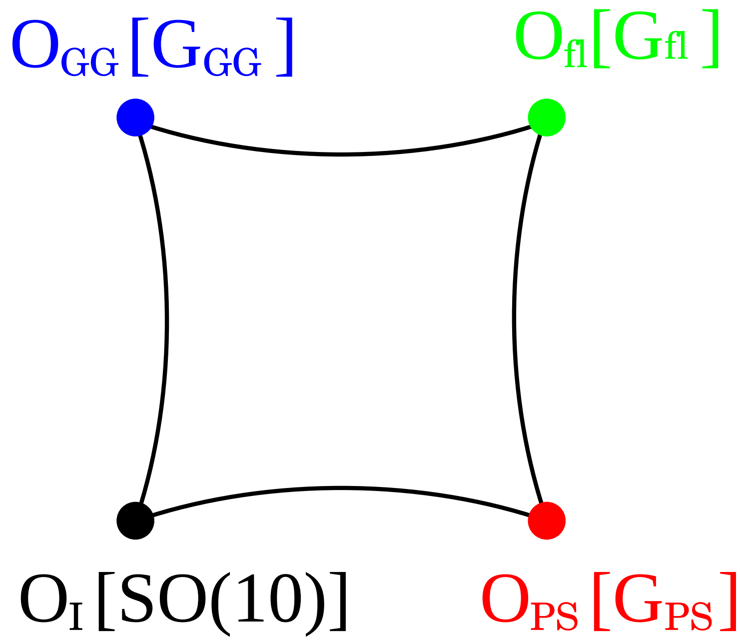
remarkably consistent with results from neutrino oscillations, $\sqrt{\Delta m^2_{atm}} \sim 0.05$ eV and $\sqrt{\Delta m^2_{sol}} \sim 0.008$ eV. Are we probing physics at the GUT scale $\Lambda_{GUT} \sim 10^{15}$ GeV ?!

GUT models in four dimensions (4D) problematic; attractive alternative: supergravity theories in five or six dimensions (simplest possibility: orbifold compactifications); GUT symmetry breaking at fixed points (4D 'branes'), yields automatically required doublet-triplet splitting of Higgs fields.

Example: SO(10) gauge theory in six dimensions; standard model gauge group from intersection of Georgi-Glashow and Pati-Salam



The breaking is localized at different points in the extra dimensions, O, O_{PS}, O_{GG}, O_{fl}, with standard model group in four dimensions,

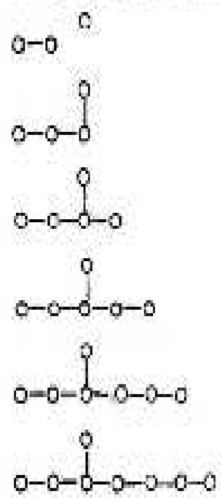


consequences: geometrical picture of flavour physics, specific predictions for proton decay modes (different from 4D),...; but only non-renormalisable effective theory...; group theory and supersymmetry lead to string theory!

Exceptional coset-spaces for quarks and leptons

he "exceptional sequence". The coset spaces $E_{n+1}/E_n \times U(1)$ of the exceptional groups contain the E_n representations which e used for quark–lepton multiplets in E_n gauge theories.

Exceptional groups	Dynkin diagrams	Coset spaces $E_{n+1}/E_n \times U(1)$
$E_3 = SU(3) \times SU(2)$		$(3,2)$ + c.c.
$E_4 = SU(5)$		10 + c.c.
$E_5 = SO(10)$		$16(=\bar{5} + 10 + 1)$ + c.c.
E_6		$27(= 16 + 10 + 1)$ + c.c.
E_7		$56(= 27 + \overline{27} + 1 + 1) + 1$ + c.c.
E_8		



interesting theoretical structure, leads to supersymmetric σ-models, relevant for some extensions of SM

Exceptional unification group $\mathrm{E}_8 \times \mathrm{E}_8$ beautifully realized in heterotic string; semi-realistic compactifications on Calabi-Yau manifolds, orbifolds...; further compactifications with Wilson lines: many models 'similar to' SM,...., but not the standard model!

Fundamental problem: huge number of vacua in string theory,

$$N_{\mathrm{vac}} = 10^X,$$

with $X = 1500$ (Lerche, Lüst, Schellekens '87) or $X = 500$ (Bousso, Polchinski '00) or ... ?

Recent, interesting approaches to find realistic string vacua involve compactifications on Calabi-Yau spaces with vector bundles, intersecting D-brane models, F-theory,...., active field of research; intermediate step of unification helpful to find realistic vacua?!

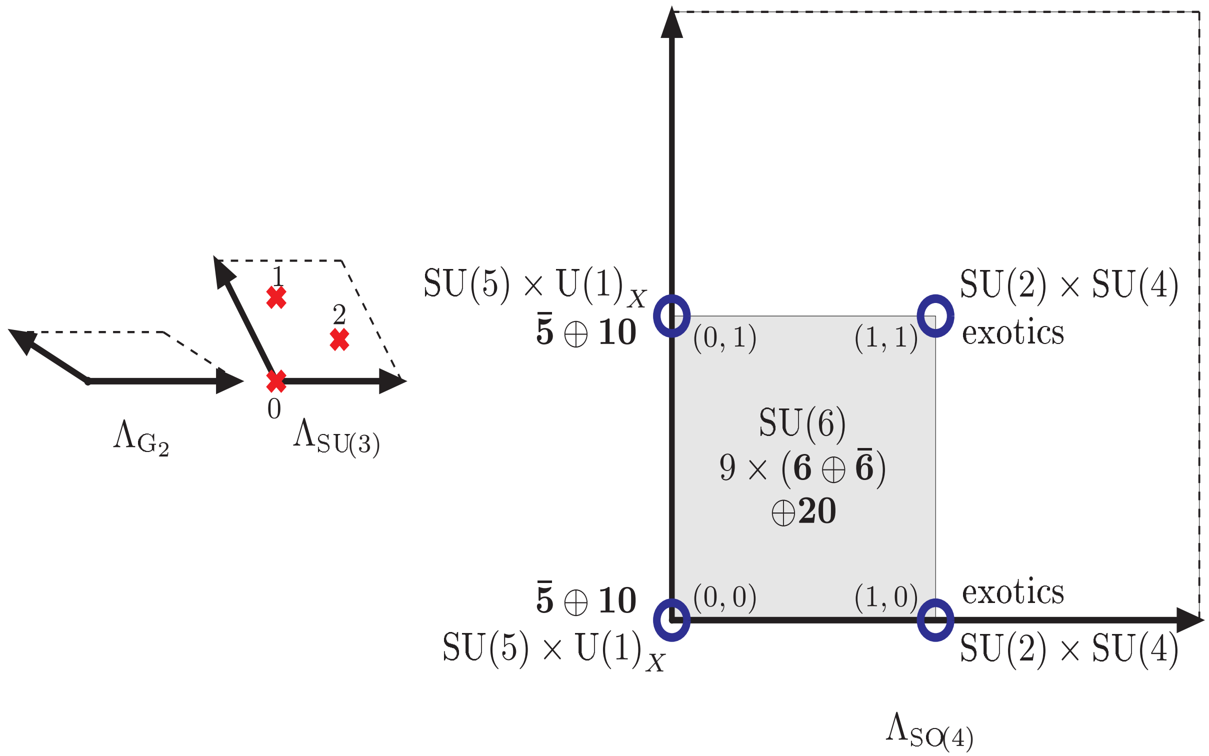
Heterotic string with local grand unification

Orbifold GUTs only effective field theories with limited predictivity, embedding in heterotic string? Compactifications on anisotropic orbifolds yields $\mathcal{O}(100)$ models with standard model gauge group and massless spectrum, without exotics: ‘mini-landscape’, considerable work of several groups during past four years.

Qualitative features:

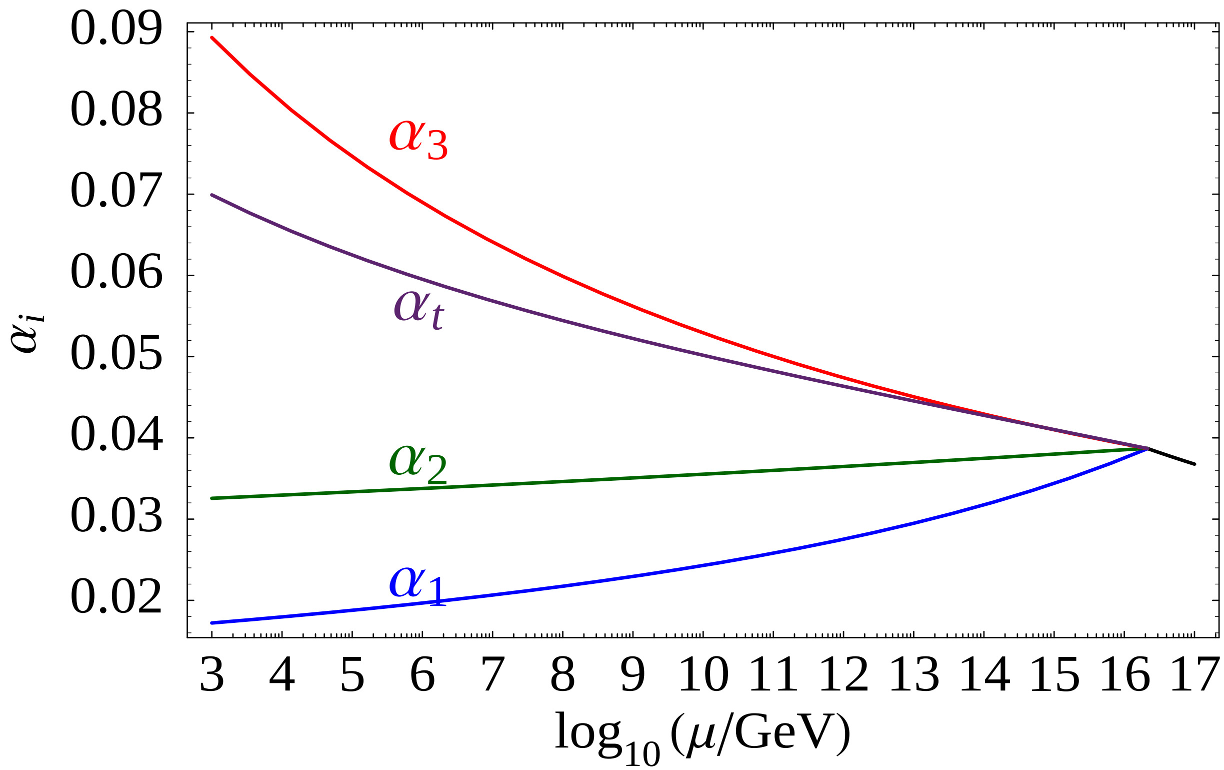
- Gauge symmetry breaking, matter and Higgs sector, and scale of supersymmetry breaking are related (big puzzle!)
- Hierarchical Yukawa couplings à la Froggatt-Nielsen
- top-quark singled out, Yukawa coupling from 6D gauge coupling, 3rd ‘family’ from ‘split multiplets’.

Heterotic $\mathrm{SU}(6)$ model in six (+ four) dimensions



Local $\mathrm{SU}(5) \times \mathrm{U}(1)_\mathrm{X}$ symmetry at GUT fixed points; matter: 2 localized families, 1 'family' from 2 split bulk families; Higgs: split bulk fields

Gauge-top Yukawa unification



Qualitative picture of 4D gauge couplings and top-Yukawa coupling, all given by 6D gauge coupling; $\alpha_i = g_i^2/(4\pi)$, $\alpha_t = Y_t^2/(4\pi)$ with $g_i = Y_t$ at the GUT scale (normalization of kinetic terms!)

Expectations on the eve of the LHC

New strong interactions at the LHC (?)

- Technicolour, composite quarks and leptons, little Higgs, littlest Higgs...

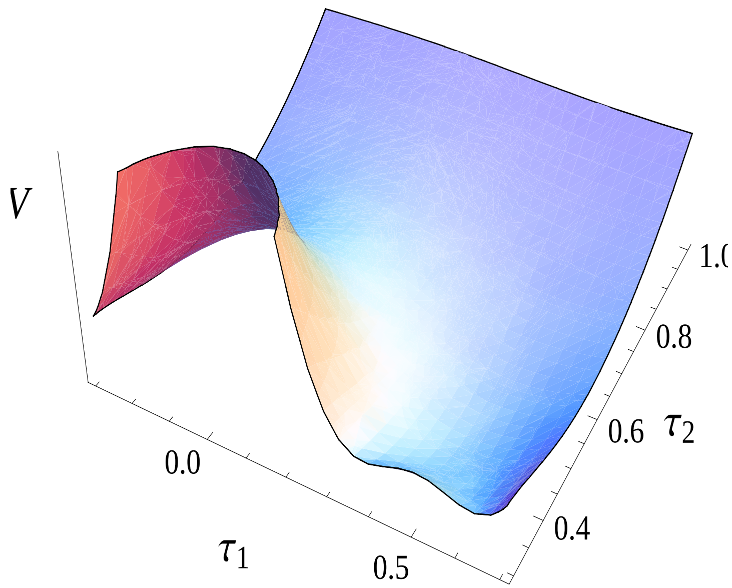
- Strong gravity at TeV scale, Randall-Sundrum scenario, large extra dimensions, 'mini-black holes', ...

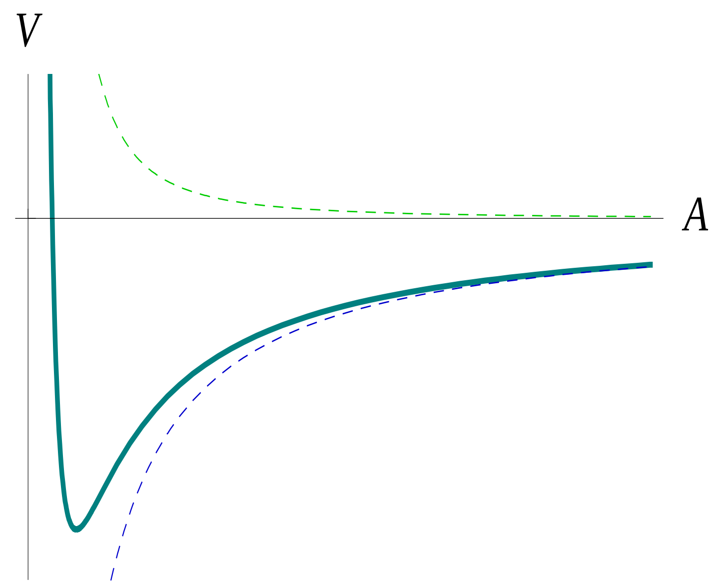
Weakly coupled theory at high energies,

- Unification, hints: symmetries and particle content of standard model, smallness of neutrino masses and seesaw mechnism, approximate unification of gauge couplings

- Supersymmetry, hints: 'precise' unification of gauge couplings, cosmologically viable candidates of dark matter (WIMP, gravitino,...)

- Small Extra Dimensions: $R \sim 1/M_{\rm GUT}$? Stabilization mechanism? Connection with supersymmetry breaking and inflation? Vacuum energy density $\rho_{\rm vac} \sim \mu^2_{\rm SUSY} M^2_{\rm GUT} \ll M^4_{\rm GUT}$? Additional singlets?





Physics at the LHC:

- Discovery of Higgs and supersymmetry

- Determination of Higgs and top masses and couplings; departures from (MS)SM?

- Discovery of LSP, consistency with Dark Matter?

- Determination of supersymmetry breaking mechanism, consistent with unification?

- Dynamics of compactification, additional singlets with masses $\mathcal{O}(m_{3/2})$, remnant of vacuum degeneracy; discovery in late decays of superparticles?