Neutrino Mass Mechanisms *and Leptogenesis*

Sacha Davidson

IPN de Lyon, IN2P3/CNRS

1. **neutrino masses (majorana or dirac) and mixing angles**
2. **mechanisms for small Dirac masses**
3. **mechanisms for small Majorana masses**
   - **suppressed by a large mass scale and small couplings: the seesaw**
   - **suppressed by small couplings and loops: R_p violation in SUSY**
4. leptogenesis
   - required ingredients for baryogenesis
   - baryogenesis via leptogenesis
   - flavoured thermal leptogenesis (type I seesaw, hierarchical N_i)

(mechanism = particle content and interactions
models restrict numerical value of coupling constants)

An overview of the history of neutrinos (hypothetical /known neutrino activities)

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-
- inflation (produce large scale CMB fluctuations) (?could be driven by the sneutrino?)
- **baryogenesis (excess of matter over anti-matter)?leptogenesis in the seesaw?**
- relic density of (cold) Dark Matter (?could be (heavy) neutrinos too??? Shaposhnikov et al)
- Big Bang Nucleosynthesis (produce $H, D,^3 He,^4 He,^7 Li$ abundances at $T \sim$ MeV))
  $\Leftrightarrow$ 3 species of relativistic ν in the thermal soup
- decoupling of photons — $e + p \to H$ (CMB spectrum today)
  cares about radiation density $\leftrightarrow N_\nu, m_\nu$
- for 10^{10} yrs —stars are born, radiate (γ, ν), and die
- supernovae explode (?thanks to ν?) spreading heavy elements
- 1930: Pauli hypothesises the "neutrino", to conserve E in $n \to p + e(+\nu)$
- 1953 Reines and Cowan: neutrino CC interactions in detector near a reactor
- invention of the Standard Model
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-
-
- REFS CAN BE FOUND AT : http://www.nu.to.infn.it/Neutrino_Models/ ...FOR INSTANCE:
- ν mass mechanisms: Mohapatra+Smirnov (ARNPS 0603118), Altarelli+Feruglio(flavour syms), Mukhopadhyaya (SUSY, 0301278), Grimus (0612311).
- ν pheno: Garcia-Gonzalez+Maltoni(PhysRep:0704.1800), Garcia-Gonzalez+Nir(RMP 0202058),
- leptogen:SDNardiNir (flav,PhysRep:0802.2962), Giudice$etal$(thermal,NPB 0310123), Buchmuller$etal$(analytic approx,"pedestrians" 0401240)

Observables: masses and a mixing matrix for three generations

Two mass differences: hierarchical ($m_1 < m_2 < m_3$), or inverse hierarchical ($m_2 > m_1 > m_3$):

$$\Delta m^2_{atm} = m_3^2 - m_2^2 \simeq 2.6 \times 10^{-3}\mathrm{eV}^2 \qquad \Delta m^2_{\odot} = m_2^2 - m_1^2 \simeq 7.9 \times 10^{-5}\mathrm{eV}^2$$

Two angles of the mixing matrix (lives in generation space. Rotates from charged lepton mass basis to neutrino mass basis). Majorana mixing matrix is U. Dirac neutrino mixing matrix is V:

$$U = V \cdot diag\{e^{-i\phi/2}, e^{-i\phi'/2}, 1\}$$

$$V_{\alpha i} = \begin{bmatrix} c_{12}c_{13} & c_{13}s_{12} & s_{13}e^{-i\delta} \\ -c_{23}s_{12} - c_{12}s_{13}s_{23}e^{i\delta} & c_{12}c_{23} - s_{12}s_{13}s_{23}e^{i\delta} & c_{13}s_{23} \\ s_{23}s_{12} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - c_{23}s_{12}s_{13}e^{i\delta} & c_{13}c_{23} \end{bmatrix}.$$

$$\theta_{23} \simeq .7 \pm .2 \simeq \pi/4 \quad \theta_{12} \simeq .6 \pm .1 \simeq \pi/6 \quad \theta_{13} \leq .2$$

δ, ϕ, ϕ' unknown —$\not{C}\not{P}$ in lepton sector not observed (yet). Exercise: count phases...

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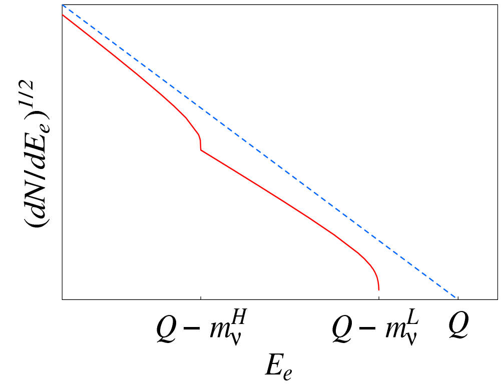
Upper bounds on the mass scale:

$$\sum_i |m_i| \leq .37 - 2 \text{ eV} \;\; \text{LSS and CMB} \qquad [m^2]_{ee} \lesssim 2 \text{ eV} \quad \beta\ decay,\ Mainz$$

$$|[m]_{ee}| \; \begin{cases} \lesssim .35 \text{ eV} & 0\nu 2\beta, HM \\ \simeq .3 \text{ eV} & 0\nu 2\beta, KK \end{cases}$$

$n \to p + e + \bar{\nu}$: m_ν distorts e spectrum

Consider Tritium β decay:
$^3H \to ^3He + e + \bar{\nu}_e \quad , \quad Q = 18.6 \text{ eV}$
where $E_e = 18.6 \text{ keV} - E_\nu \leq 18.6 \text{ keV} - $ "$m_{e\nu}$"
Endpoint of e spectrum :
$\frac{dN_e}{dE_e} \propto \sum_i |U_{ei}|^2 \sqrt{(18.6 \text{ keV} - E_e)^2 - m_{\nu_i}^2}$



Current bound: $m_{\nu e} \lesssim 2$ eV Katrin sensitivity ~ 0.3 eV.

http://www-ik.fzk.de/tritium/

Exercise: *how to detect CNB?*

In the room, are $\sim 10^6$ WIMPS, $\sim 10^5$ Be ν, and $\sim 10^{10}$ Cosmic Background Neutrinos(CNB).

What about ν capture βdecay: $n + \nu_{CNB} \rightarrow p + e$?

Cocco Mangano Messina

To compare rate for ${}^3H \rightarrow {}^3 He + e + \bar{\nu}_e$ to $\nu_e + {}^3 H \rightarrow {}^3 He + e$:

$$\frac{n_{\nu CNB}}{\nu \text{ phase space}} \simeq \frac{T^3_{CNB}}{\pi^2} \frac{1}{Q^3} \sim \left(\frac{10^{-4}\text{eV}}{20\text{keV}} \right)^3 \sim 10^{-24}$$

But...$E_e = Q + m_\nu$
(recall for ${}^3H \rightarrow {}^3 He + e + \bar{\nu}_e$, $E_e \leq Q - m_\nu$)

So...if ever resolution better than m_ν...

helicity, chirality and all that...

ψ a Dirac spinor, 4 degrees of freedom labelled by $\{\pm E, \pm s\}$.

Chiral decomposition of $\psi \; = \; \psi_L \; + \; \psi_R$,

$$\psi_L = \; P_L \, \psi \quad \text{avec} \quad P_L = \frac{(1-\gamma_5)}{2} \qquad , \qquad \psi_R = \; P_R \, \psi \quad \text{avec} \quad P_R = \frac{(1+\gamma_5)}{2}$$

not an observable; property of the field ($P_{L,R}$ simple to calculate with :))
independent of reference frame—but becomes helicity in the relativistic limit.
Standard Model is chiral $=$ different gauge interactions for LH, RH fermions.

define **helicity** as $\pm \hat{s} \cdot \hat{k} = \pm 1/2$, for particle of 4-momentum $(k_0, \vec{k})$. Observable. Ugly operator.

Gauge kinetic terms for chiral fermions : $\overline{\psi} \, \gamma^\mu D_\mu \psi = \overline{\psi_L} \, \gamma^\mu D_\mu \psi_L \; + \overline{\psi_R} \, \gamma^\mu D_\mu \psi_R$,

but not the Dirac mass: $m \overline{\psi} \, \psi \; = \; m \overline{\psi_L} \, \psi_R \; + m \overline{\psi_R} \, \psi_L$

Careful about notation: $\overline{(\psi_R)} = (\overline{\psi})_L \neq (\overline{\psi})_R$

To write a mass for ν_L... Dirac or Majorana

Work in effective theory of SM below m_W. SU(2) (spontaneously) broken, so a mass term for ν_L is allowed. It must be Lorentz invariant. Allowed mass term, four-component fermion ψ:

$$m\overline{\psi}\,\psi \;=\; m\overline{\psi_L}\,\psi_R \;+ m\overline{\psi_R}\,\psi_L$$

1. **Dirac masss term**:
   SM has only ν_L, 2 dof(degree of freedom) chiral fermion $\Rightarrow$ introduce another 2 dof chiral gauge singlet fermion ν_R
   Construct fermion number conserving mass term like all other SM fermions:

   $$m\overline{\nu_L}\,\nu_R \;+ m\overline{\nu_R}\,\nu_L$$

   In full SM: $\lambda(\overline{\nu_L}, \overline{e_L}) \begin{pmatrix} H_0 \\ -H_+ \end{pmatrix} \nu_R \equiv \lambda(\overline{\ell} H) e_R \to m\overline{\nu_L}\,\nu_R \quad , \quad m = \lambda\langle H_0\rangle$

2. **Majorana mass term**: the charge conjugate of ν_L is right-handed ! Exercise: check this.
   $\Rightarrow$ can write a fermion number non-conserving mass term using just 2 dof of ν_L.
   No new fields, but lepton number violating mass.
   With multiple generations, $[m]_{\alpha\beta}$ will be a *symmetric* matrix Exercise: check this.

   In full SM: $\frac{K}{M}(\ell H)(\ell H) \to m\nu_L\nu_L \quad , \quad m = \frac{K}{M}\langle H_0\rangle^2$

Majorana mass term: the charge conjugate of ν_L is right-handed

$$\psi = \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix} \quad , \quad \{\gamma^\alpha\} = \left\{ \begin{bmatrix} 0 & I \\ I & 0 \end{bmatrix}, \begin{bmatrix} 0 & \sigma_i \\ -\sigma_i & 0 \end{bmatrix} \right\}$$

$$\{\sigma_i\} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$\psi^c = -i\gamma_0\gamma_2\bar{\psi}^T = -i\gamma_0\gamma_2\gamma_0\psi^* = i\gamma_2^*\psi^* = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \\ 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \begin{pmatrix} \begin{pmatrix} \psi_L^* \end{pmatrix} \\ \begin{pmatrix} \psi_R^* \end{pmatrix} \end{pmatrix}$$

$$(\nu_L)^c = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \\ 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \begin{pmatrix} \begin{pmatrix} \nu_L^* \end{pmatrix} \\ \begin{pmatrix} 0 \\ 0 \end{pmatrix} \end{pmatrix} = \begin{pmatrix} \begin{pmatrix} 0 \\ 0 \end{pmatrix} \\ \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \begin{pmatrix} \nu_L^* \end{pmatrix} \end{pmatrix} = \begin{pmatrix} \begin{pmatrix} 0 \end{pmatrix} \\ \begin{pmatrix} -i\sigma_2\nu_L^* \end{pmatrix} \end{pmatrix}$$

Allowed mass term, four-component fermion ψ: $m\overline{\psi}\,\psi \;=\; m\overline{\psi_L}\,\psi_R \;+ m\overline{\psi_R}\,\psi_L$

$\Rightarrow$ with *only* the 2 dof of a chiral fermion, can write mass term:

$$\begin{aligned} \frac{m}{2}[\overline{\nu_L}(\nu_L)^c + \overline{(\nu_L)^c}\nu_L] &= \frac{m}{2}[(\nu_L)^\dagger\gamma_0(\nu_L)^c + ((\nu_L)^c)^\dagger\gamma_0\nu_L] = -i\frac{m}{2}[\nu_L^\dagger\sigma_2\nu_L^* + \nu_L^T\sigma_2\nu_L] \\ &\equiv \frac{m}{2}\nu_L\nu_L + h.c. \end{aligned}$$

($1/2$ in Lagrangian is like for real scalar masses)

Majorana mass matrix is symmetric

Can write a majorana mass term (one generation) as

$$\frac{1}{2}m[\overline{\nu_L}(\nu_L)^c + \overline{(\nu_L)^c}\nu_L] = \frac{-im}{2}[\nu_L^\dagger\sigma_2\nu_L^* + \nu_L^T\sigma_2\nu_L] = \frac{m}{2}\nu_L\nu_L + h.c.$$

With multiple generations, $[m]_{\alpha\beta}$ will be a *symmetric* matrix:

$$\frac{1}{2}\nu_{L\alpha}[m]_{\alpha\beta}\nu_{L\beta} + h.c. = \frac{1}{2}\nu_{L\alpha}[U^*U^TmUU^\dagger]_{\alpha\beta}\nu_{L\beta} + h.c. = \frac{1}{2}\nu_{Li}m_i\nu_{Li} + h.c.$$

Yes! fermion fields anti-commute. But for ρ, σ spinor indices, $\nu_{Li}^\rho\varepsilon_{\rho\sigma}\nu_{Lj}^\sigma = -\nu_{Lj}^\sigma\varepsilon_{\rho\sigma}\nu_{Li}^\rho = \nu_{Lj}^\sigma\varepsilon_{\sigma\rho}\nu_{Li}^\rho$ $mm^\dagger$ hermitian, obtain U from $U^Tmm^\dagger U^* = D_m^2$.
U called PMNS matrix (for Pontecorvo, Maki, Nakagawa and Sakata) : U_{PMNS}.

reminder about the Dirac mass matrix (if added 3 ν_R to the SM): arbitrary 3 $\times$ 3 matrix (like other SM Yukawa couplings). In charged lepton mass eigenstate basis for $\nu_L \equiv$ " flavour basis" (indices $\alpha, \beta\ldots$), diagonalise with independent transformations on SU(2) doublet/singlet indices:

$$\overline{\nu_L}_\alpha[m]_{\alpha b}\nu_{Rb} + \overline{\nu_R}_b\,[m]^*_{b\alpha}\nu_{L\alpha} = \overline{\nu_L}_\alpha[V_L^*V_L^TmV_R^*V_R^T]_{ab}\nu_{Rb} + h.c = \overline{\nu_L}_j\,m_j\nu_{Rj} + h.c.$$

$mm^\dagger$ hermitian, obtain V_L from $V_L^Tmm^\dagger V_L^* = D_m^2$. (real eigenvals for hermitian matrices).

counting mixing matrix phases: 1 for Dirac, 3 for Majorana

- A 3×3 complex matrix has 18 real parameters
- the unitarity condition $VV^\dagger = 1$, $UU^\dagger = 1$ reduces this to 9, which can be taken as 3 angles and 6 phases.
- five of those phases are relative phases between the fields $e, \mu, \tau, \nu_1, \nu_2$ and ν_3
- ...so if we are free to choose the phases of all the LH fermions, we are left with one phase in the mixing matrix. This is the case for a dirac mass matrix ($e.g.$ quarks), where any potential phase on the masses could be absorbed by the RH fermion fields. Also the case in oscillations, where appears $mm^\dagger$.
- if ν_L have Majorana masses, between themselves and their antiparticle, it is the LH neutrino field which must absorb the phase off the Majorana mass. So in physical processes where the Majorana mass appears linearly (not as mm^*; eg $0\nu 2\beta$), one can choose the phase such that the mass is real—in which case one can remove one less phase from MNS, or one can keep MNS with one phase, and allow complex masses.
- it is always possible to remove the phase from one majorana mass, by using the global overall phase of all the leptons (the sixth phase of $e, \mu, \tau, \nu_1, \nu_2$ and ν_3, which we could not use to remove phases from the lepton number conserving PMNS matrix). So in three generations, there are possibly two complex majorana neutrino masses, so two “Majorana” phases in addition to the “Dirac” phase δ of PMNS.

Recall...upper bound on m_ν from $0\nu 2\beta$

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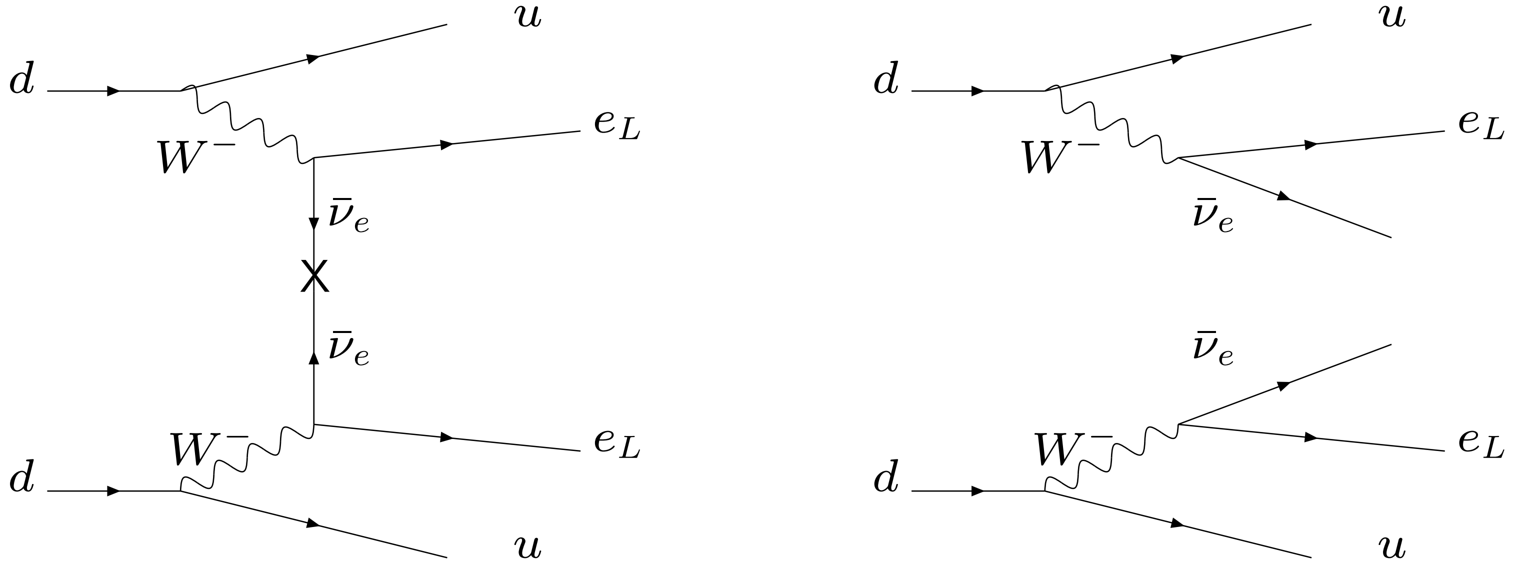
Neutrino*less* double beta decay: $(Z, A) \rightarrow (Z+2, A) + 2e$

Single β decay kinematically forbidden for some nuclei
(eg ${}^{76}_{32}Ge$ lighter than ${}^{76}_{33}As$, so ${}^{76}_{32}Ge \rightarrow {}^{76}_{34}Se + ee\bar{\nu}_e\bar{\nu}_e$. $\tau \sim 10^{21}$ yrs)



only for majorana neutrinos...

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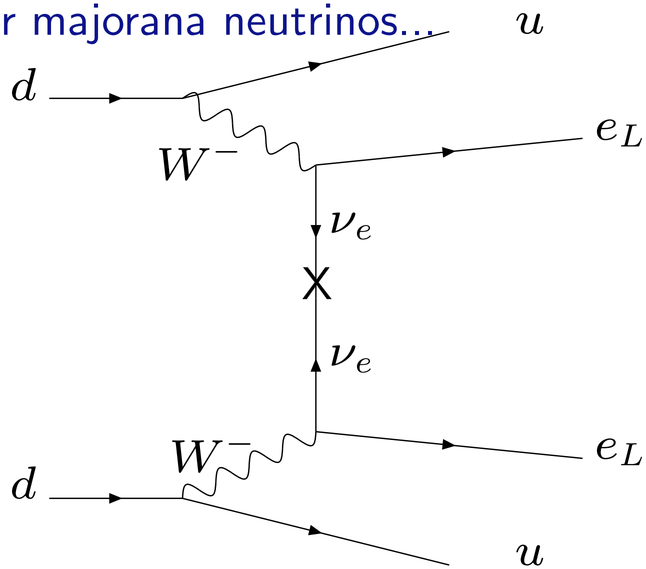
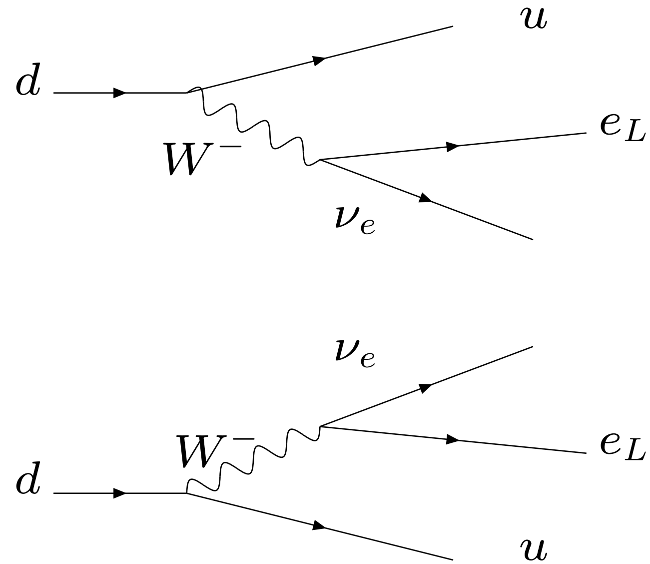
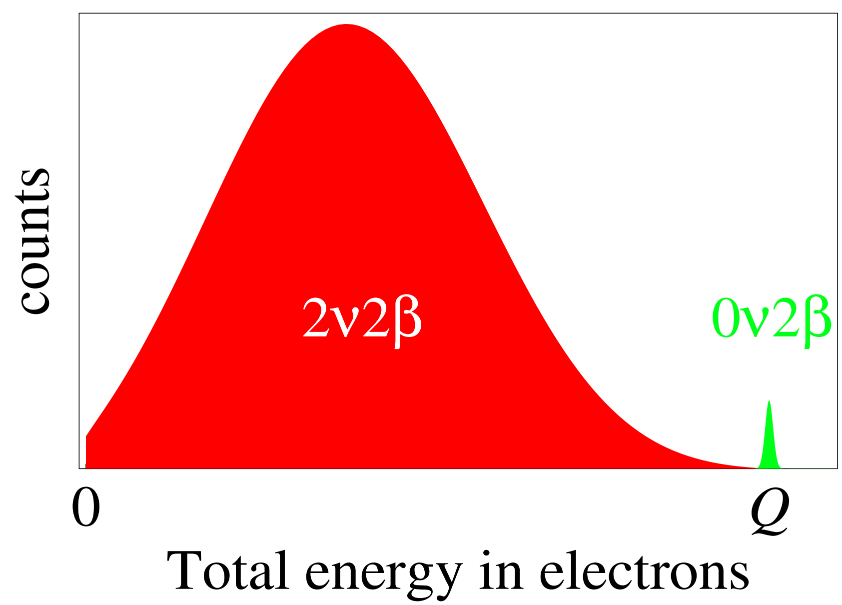
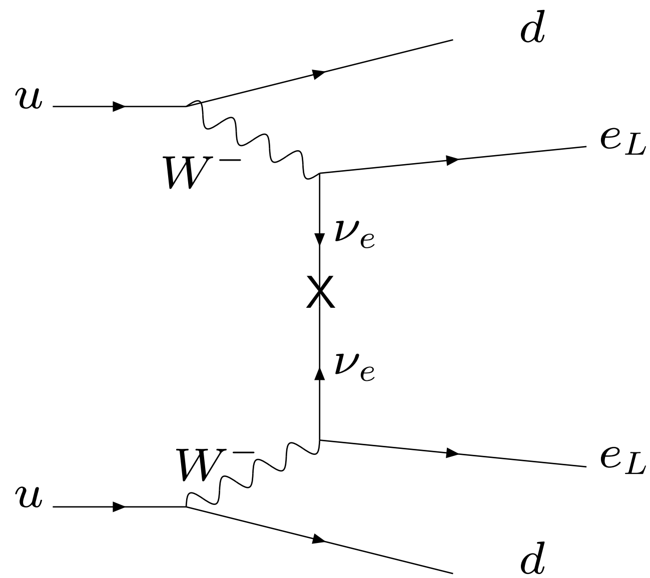






$0\nu2\beta$—what can we learn?



$$|\mathcal{M}|^2 = \left| \begin{array}{c} \text{nuclear} \\ \text{matrix} \\ \text{element} \end{array} \right|^2 \times \left| \sum_i U_{ei}^2 m_i \right|^2$$

$$|\mathcal{M}|^2 \quad \propto \quad \left| c_{13}^2 c_{12}^2 e^{-i2\phi} m_1 + c_{13}^2 s_{12}^2 e^{-i2\phi'} m_2 + s_{13}^2 e^{-i2\delta} m_3 \right|^2$$

... appearance of the majorana phases!
but: $\propto m_\nu^2$, and ± 3? from nuclear matrix element

(Exercise: find other processes sensitive to other majorana masses. Publish if they could be measured in your lifetime.)

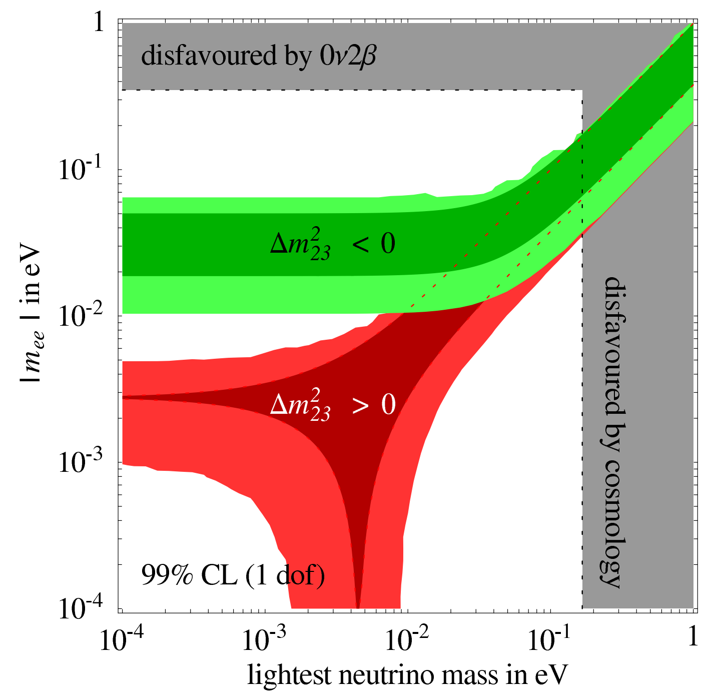
What can we learn?

$$
\begin{aligned}
|\mathcal{M}|^2 &\propto \left| c_{13}^2 c_{12}^2 e^{-i2\phi} m_1 + c_{13}^2 s_{12}^2 e^{-i2\phi'} m_2 + s_{13}^2 e^{-i2\delta} m_3 \right|^2 \\
&\propto |\frac{3}{4} e^{-i2\phi} m_1 + \frac{1}{4} e^{-i2\phi'} m_2 + s_{13}^2 e^{-i2\delta} m_3|^2 \\
&\to |\frac{3}{4} e^{-i2\phi} m_1 + \frac{1}{4} e^{-i2\phi'} m_{sol} + < (.2)^2 e^{-i2\delta} m_{atm}|^2 \simeq m_{sol}^2 |\frac{3m_1}{m_{sol}} + e^{-i2(\phi-\phi')}|^2 \\
&\to m_{atm}^2 |3 + e^{-i2(\phi'-\phi)}|^2
\end{aligned}
$$

Determine mass hierarchy at a ν beam.

• Inverse hierarchy ($m_1 \sim m_2 > m_3$):
observe at $|m_{ee}| \sim m_{atm}$,
OR neutrinos are Dirac

• Hierarchical ($m_1 < m_2 < m_3$):
observe at $|m_{ee}| \sim m_{sol}$, if m_1 negligeable,
BUT can vanish for $m_1 \sim m_{sol}/3$



Dirac masses

Puzzle 1: if the observed neutrino masses are Dirac : $m\overline{\nu_L}\nu_R + hc$, why are neutrino Yukawa eigenvalues $\ll$ other fermions?

- in SUSY, put a symmetry to forbid as an F-term. Appears in SUGRA as a D-term $\propto m_{SUSY}/m_{pl}$. ArkaniHamed Borzumati
- in extra dimensions, ν_R and ν_L live in different places: little overlap. Grossman+Neubert...
- Ignore this puzzle: we don't understand Yukawas
- ...

Puzzle 2: ν_R is gauge singlet, why does it not have a majorana mass? (not forbidden by SM gauge symmetries...)

- Put a symmetry. Such as lepton number L, or $B - L$.

Outline (again)

1. neutrino masses (majorana or dirac) and mixing angles
2. mechanisms for small Dirac masses
3. **mechanisms for small Majorana masses**
   - **suppressed by a large mass scale and small couplings: the seesaw**
   - suppressed by small couplings and loops: R_p violation in SUSY
4. leptogenesis
   - required ingredients for baryogenesis
   - baryogenesis via leptogenesis
   - flavoured thermal leptogenesis type I seesaw, hierarchical N_i

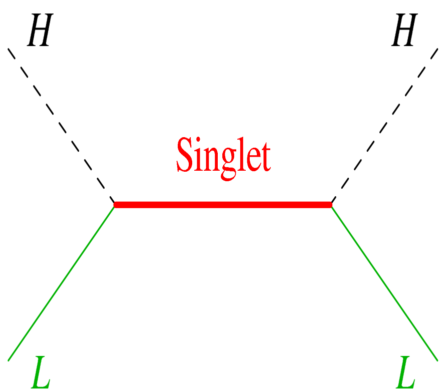
Tree-level exchange of a heavy particle $=>$ small m_ν:

Want heavy new particles (mass M), which induce dimension 5 effective operator :

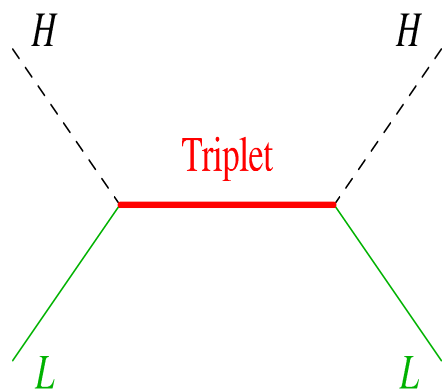
$$\frac{1}{M}[\ell H][\ell H] \to \nu\nu\frac{\langle H_0\rangle^2}{M}$$

Three possibilities:

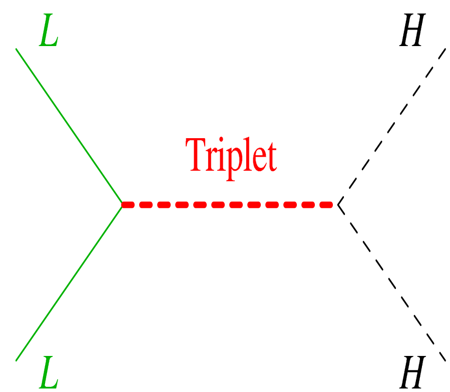
SU(2) singlet fermions
Type I



SU(2) triplet fermions
Type III



SU(2) triplet scalars
Type II



Neutrino Masses (in the one generation seesaw)

Adding sterile N, Yukawa coupling, and Majorana mass for N:

$$\mathcal{L}_{lep}^{Yuk} = -h_e(\overline{\nu_L}, \overline{e_L}) \begin{pmatrix} H^+ \\ -H^0 \end{pmatrix} e_R \; + \; \lambda(\overline{\nu_L}, \overline{e_L}) \begin{pmatrix} H^0 \\ -H^- \end{pmatrix} N \; + \; M\overline{N^c}N$$

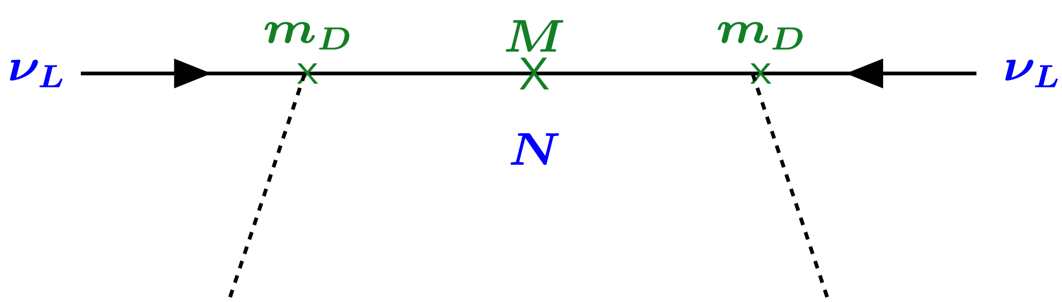
$$m_e \bar{e}_L e_R \qquad\qquad +m_D \bar{\nu}_L N \qquad\qquad +M\overline{N^c}N$$

$\Rightarrow$ neutrino mass matrix:

$$\begin{pmatrix} \bar{\nu}_L & \overline{N^c} \end{pmatrix} \begin{bmatrix} 0 & m_D \\ m_D & M \end{bmatrix} \begin{pmatrix} \nu_L^c \\ N \end{pmatrix}$$

$(\nu_L^c \equiv (\nu_L)^c)$

eigenvectors $\simeq \nu_L$ with $m_\nu \sim \frac{m_D^2}{M}$ $\qquad$ N with mass $\sim M$

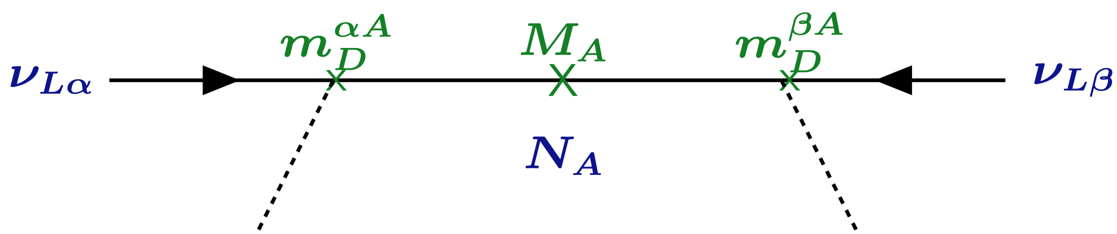


The See-Saw in three generations

- in the charged lepton ("flavour") and $N (= \nu_R)$ mass bases, at large energy scale $\gg M_i$:

21 parameters chez les leptons:
$m_e, m_\mu, m_\tau, M_1, M_2, M_3$
18 - 3 (ℓ phases) in λ

$$\mathcal{L} \quad = \quad \mathcal{L}_{SM} + \lambda^*_{\alpha J} \overline{\ell}_\alpha \cdot H N_J - \frac{1}{2} \overline{N_J} M_J N_J^c$$



- at the weak scale, get effective light neutrino mass matrix

12 parameters:
$m_e, m_\mu, m_\tau, m_1, m_2, m_3$
6 in U_{MNS}

$$\lambda M^{-1} \lambda^{\mathbf{T}} \langle H^0 \rangle^2 \quad = \quad [m_\nu] \quad = U^* D_m U^\dagger$$

Tangent—diagonalising a Majorana mass matrix

To find eigenvectors $\vec{v}_i$ of a hermitian matrix A, with eigenvalues $\{a_i\}$

$$A\vec{v}_i = a_i\vec{v}_i$$

For Majorana matrix :

$$A\vec{u}_i = a_i\vec{u}_i^*$$

hermitian : $V^\dagger AV = D_A = diag\{a_1, ...a_n\}$ (V unitary)

$$\begin{bmatrix} & & \\ & A & \\ & & \end{bmatrix}\left[\begin{pmatrix} \\ \vec{v}_1 \\ \\ \end{pmatrix}\begin{pmatrix} \\ \vec{v}_2 \\ \\ \end{pmatrix}\begin{pmatrix} \\ \vec{v}_3 \\ \\ \end{pmatrix}\right] = \left[\begin{pmatrix} \\ \vec{v}_1 \\ \\ \end{pmatrix}\begin{pmatrix} \\ \vec{v}_2 \\ \\ \end{pmatrix}\begin{pmatrix} \\ \vec{v}_3 \\ \\ \end{pmatrix}\right]\begin{bmatrix} a_1 & & \\ & \ldots & \\ & & a_n \end{bmatrix}$$

majorana : $U^TAU = D_A \Rightarrow AU = U^*D_A$ (U unitary $UU^\dagger = 1$)

$$\begin{bmatrix} & & \\ & A & \\ & & \end{bmatrix}\left[\begin{pmatrix} \\ \vec{u}_1 \\ \\ \end{pmatrix}\begin{pmatrix} \\ \vec{u}_2 \\ \\ \end{pmatrix}\begin{pmatrix} \\ \vec{u}_3 \\ \\ \end{pmatrix}\right] = \left[\begin{pmatrix} \\ \vec{u}_1^* \\ \\ \end{pmatrix}\begin{pmatrix} \\ \vec{u}_2^* \\ \\ \end{pmatrix}\begin{pmatrix} \\ \vec{u}_3^* \\ \\ \end{pmatrix}\right]\begin{bmatrix} a_1 & & \\ & \ldots & \\ & & a_n \end{bmatrix}$$

And another curiosity about diagonalisation...

A hermitian matrix with degenerate eigenvalues is always diagonal. Not true for majorana mass matrix (due to phases on masses): its not the same to diagonalise $M^\dagger M = V^\dagger D_M^2 V$, or $M = U^T D_M U$ (for degenerate eigenvalues) Ex:

$$M = \left[\begin{array}{cc} 0 & M_1 e^{i\phi} \\ M_1 e^{i\phi} & 0 \end{array} \right] \; , \qquad M^\dagger M = \left[\begin{array}{cc} M_1^2 & 0 \\ 0 & M_1^2 \end{array} \right] \qquad M_1 \in \Re$$

Outline (again)

1. neutrino masses (majorana or dirac) and mixing angles

2. mechanisms for small Dirac masses

3. **mechanisms for small Majorana masses**

   - suppressed by a large mass scale and small couplings: the seesaw
   - **suppressed by small couplings and loops: R_p violation in SUSY**

4. leptogenesis

   - required ingredients for baryogenesis
   - baryogenesis via leptogenesis
   - flavoured thermal leptogenesis type I seesaw, hierarchical N_i

Small m_ν from small couplings and loops: RPV SUSY

Summary: in supersymmetric theories with R-parity (lepton number) violation (RPV), neutrino masses can arise at tree level and at one-loop.

In the SM, the Higgs and leptons have the same gauge quantum numbers— but cannot confuse a scalar with a fermion.

In SUSY, only difference between slepton $\begin{pmatrix} \tilde{\nu} \\ \tilde{e}_L \end{pmatrix}$ and Higgs $H_d = \begin{pmatrix} H_0 \\ H_- \end{pmatrix}$ is lepton number

$\Leftrightarrow$ if L not conserved, can replace $\ell \leftrightarrow \tilde{h}_d$, $H_0 \leftrightarrow \tilde{\nu}$:

$$\begin{array}{cccc}
\text{SUSY with L cons} & \rightarrow & \text{SUSY with L NOT cons.} & \text{Superpotential} \\
\mu[\tilde{h}_d \tilde{h}_u] & \rightarrow & \epsilon_\alpha[\ell_\alpha \tilde{h}_u] \; forget\ this & \epsilon_\alpha[L_\alpha H_u] \\
\mathbf{h}^e_\alpha[\ell_\alpha H_d](e_{R\alpha})^c & \rightarrow & \lambda_{\alpha\beta\rho}[\ell_\alpha \tilde{\ell}_\beta](e_{R\rho})^c & \lambda_{\alpha\beta\rho}[L_\alpha L_\beta]E^c_\rho \\
\mathbf{h}^d_\alpha[q_\alpha H_d](d_{R\alpha})^c & \rightarrow & \lambda'_{\alpha\beta\rho}[q_\alpha \tilde{\ell}_\beta](d_{R\rho})^c & \lambda'_{s\alpha t}[Q_s L_\alpha]D^c_t
\end{array}$$

where **h** SM Yukawa coupling, $H_i(\tilde{h}_i)$ the MSSM Higgses (higgsinos), [...] SU(2) weak contraction

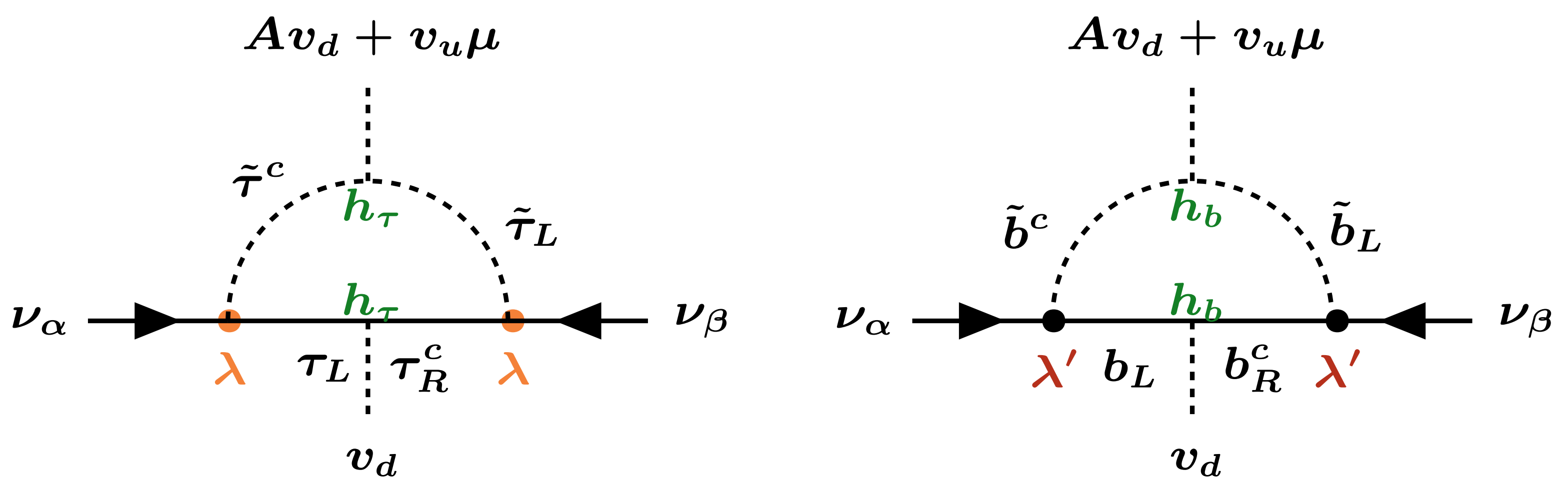
In SUSY, if *not* impose lepton number conservation, can have *renormalisable* lepton number violating interactions, constrained by contributions to m_ν, FCNC, etc. Also make LSP decay, and can put renorm B violation that allows proton decay.

m_ν in RPV —diagrams

Consider lepton number violating interactions:

$$\lambda^\tau_{\alpha\tau}[\nu_\alpha\tilde{\tau}_L](\tau_R)^c + \lambda^\tau_{\alpha\tau}[\nu_\alpha\tau_L](\tilde{\tau}_R)^c + \lambda'^b_{b\alpha}[\tilde{b}_L\nu_\alpha](b_R)^c + \lambda'^b_{b\alpha}[b_L\nu_\alpha](\tilde{b}_R)^c$$

One-loop contributions to $[\ell_\alpha H_u][\ell_\beta H_d^*]$, $[\ell_\alpha H_d^*][\ell_\beta H_d^*] \to [m_\nu]\nu_{L\alpha}\nu_{L\beta}$:



$$[m_\nu]_{\alpha\beta} \simeq \frac{\lambda^\tau_{\alpha\tau}\lambda^\tau_{\beta\tau}}{16\pi^2}\frac{m_\tau^2(A+\mu\cot\beta)}{m^2_{SUSY}} + \frac{3\lambda'^b_{b\alpha}\lambda'^b_{b\beta}}{16\pi^2}\frac{m_b^2(A+\mu\cot\beta)}{m^2_{SUSY}}$$

For afficionados: note that RPV generates D-terms like $\ell H_u \ell H_d^*$, not F-term $\ell H_u \ell H_u$

Questions

- Unknowns of the (active) neutinos: 1 angle, 1-3 phases, mass pattern+scale
  - Are neutrinos Majorana or Dirac? ** lepton number violation*!*
    $0\nu 2\beta$
  - Is there CP violation in the leptons? ** not just chez les quarks?**
    beams(T2K, next generation...)
  - What is θ_{13}?
    $\mathcal{P}_{e\to e}$ at reactors (Double CHOOZ, Daya Bay...), $\mathcal{P}_{\mu\to e}$ at beams (T2K,...)
  - What is the mass scale? (and the mass pattern)
    $0\nu 2\beta$, cosmology, ^3H decay,... (beams)
- Neutrinos masses are first evidence of BSM in the lab. Relations to BSM searches in not-neutrino experiments?
  - Neutrinos physics meet the LHC?
    Above m_W: electroweak doublets $\Rightarrow$ anything ν_L can do, e_L can do too
  - What about other precision lepton physics ($\mu \to e\gamma$, $\tau \to \ell\gamma$...)