Neutrino oscillations and supernovae

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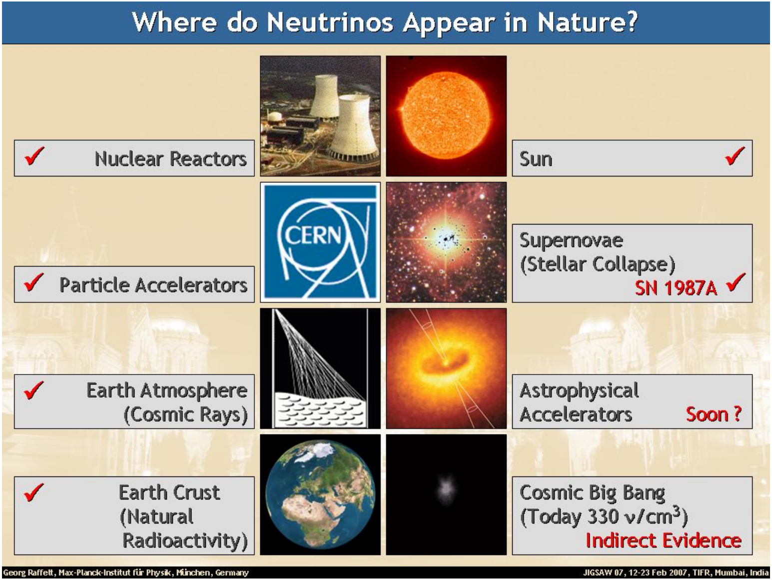
Contents

Neutrino sources



Where do Neutrinos Appear in Nature?

✓ Nuclear Reactors

Sun ✓

✓ Particle Accelerators

Supernovae
(Stellar Collapse)
SN 1987A ✓

✓ Earth Atmosphere
(Cosmic Rays)

Astrophysical
Accelerators Soon ?

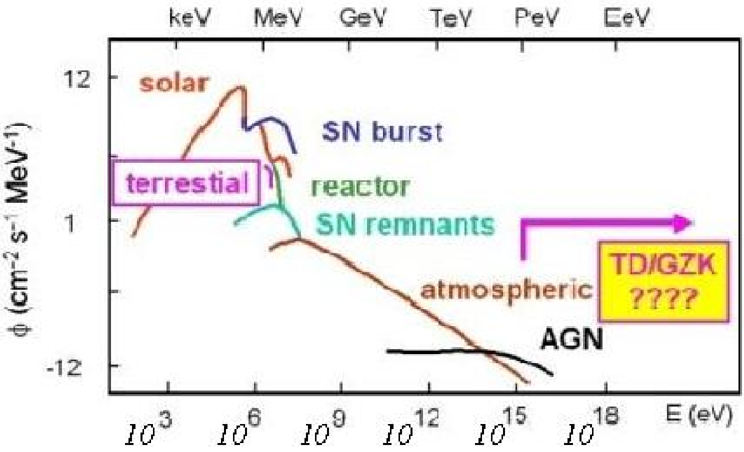
✓ Earth Crust
(Natural
Radioactivity)

Cosmic Big Bang
(Today 330 ν/cm^3)
Indirect Evidence

Georg Raffelt, Max-Planck-Institut für Physik, München, Germany

JIGSAW 07, 12-23 Feb 2007, TIFR, Mumbai, India

Spectrum of neutrino sources



Some numbers to remember

- Solar ν flux : $\sim$ 66 billion / cm^2 / sec $\sim 10^{12}/\mathrm{palm}/sec$
- Thickness of lead shielding needed to stop solar neutrinos: 100 light years
- Size of Super-Kamiokande: 40 kiloton $\sim$ this hall, with about 10 times the height
- Number of neutrinos (solar + atmospheric) detected at SK per day: $\lesssim 10$

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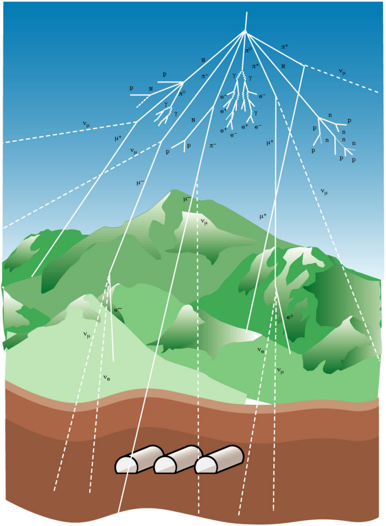
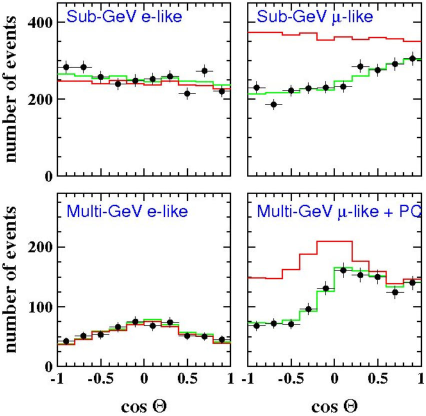
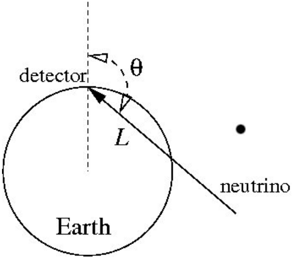
Neutrinos from cosmic rays



- $\pi^+ \rightarrow \mu^+ + \nu_\mu$
- $\mu^+ \rightarrow e^+ + \nu_e + \bar{\nu}_\mu$
- "ν_μ" flux $= 2\times$ "ν_e" flux
- "Down" flux = "Up" flux

Zenith angle dependence





Super-Kamiokande

Missing muon neutrinos !

Solution through “vacuum oscillations”

- $H = \sqrt{p^2 + m^2} \approx p + m^2/(2E)$
- Effective Hamiltonian (2 × 2):

$$H = \frac{1}{2E} \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} m_1^2 & 0 \\ 0 & m_2^2 \end{pmatrix} \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$$

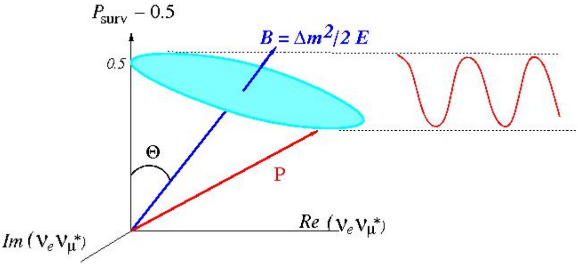
$$= \frac{m_2^2 + m_1^2}{2E} + \frac{1}{4E} \begin{pmatrix} -\Delta m^2 \cos 2\theta & \Delta m^2 \sin 2\theta \\ \Delta m^2 \sin 2\theta & \Delta m^2 \cos 2\theta \end{pmatrix}$$

- Eigenvalues: $\frac{m_1^2}{2E}, \frac{m_2^2}{2E}$
- Survival probability

$$P(\nu_\mu \to \nu_\mu) = 1 - \sin^2 2\theta \sin^2 \left(\frac{\Delta m^2 L}{4E} \right)$$

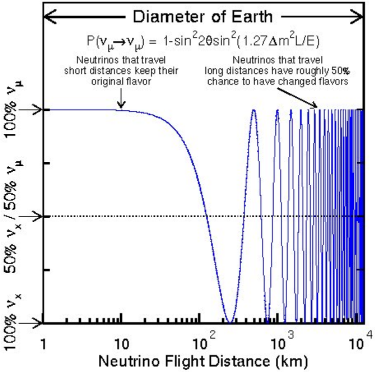
$\Delta m^2 \equiv m_2^2 - m_1^2$

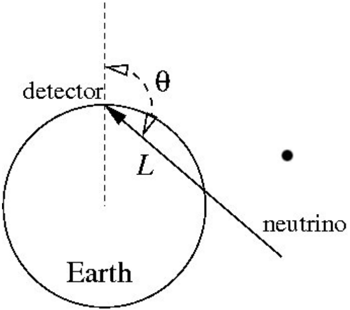
Precession of the polarization vector



- Density matrix $\rho = P_0/2 + \vec{P} \cdot \vec{\sigma}$
- Half-angle of precession $= \theta =$ mixing angle
- Different energies: same cone, different precession speeds

Solution of the atmospheric neutrino puzzle





- ν_μ oscillate into ν_τ
- ν_e do not participate

$$\Delta m^2_{\text{atm}} \approx (1.3\text{–}3.4) \times 10^{-3}\ \text{eV}^2$$

Mixing angle $\theta_{\text{atm}} \approx 36^\circ\text{–}54^\circ$

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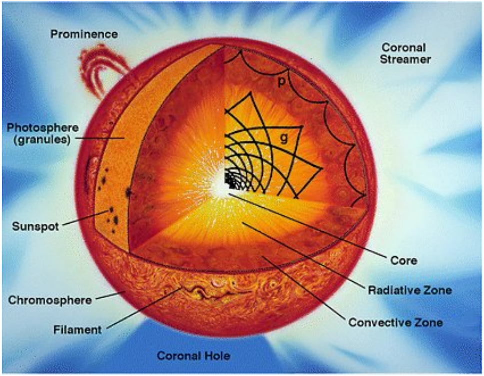
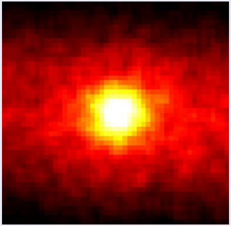
How the Sun shines



- Nuclear fusion reactions: mainly $4\,^1_1\mathrm{H} \rightarrow\,^4_2\,\mathrm{He} + 2e^+ + 2\nu_e$
- Light cannot be produced unless neutrinos are produced !!
- Davis-Koshiba Nobel prize 2002

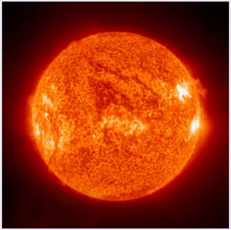
Sun: now and then

Sun in neutrinos: 8 minutes ago



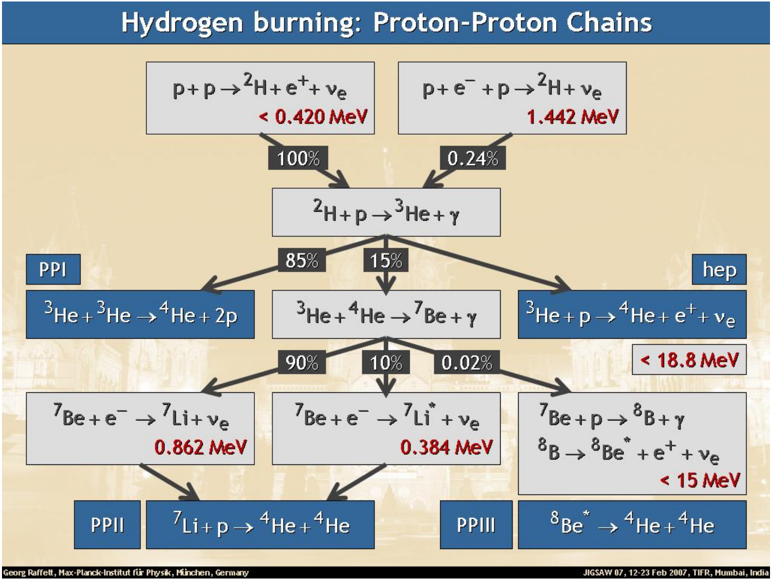
Angular size $\sim 20^\circ$

Sun in photons: a few million years ago



Angular size $\sim 1^\circ$

Nuclear reactions inside the Sun



Hydrogen burning: Proton-Proton Chains

$p + p \rightarrow {}^{2}H + e^{+} + \nu_e$ < 0.420 MeV

$p + e^{-} + p \rightarrow {}^{2}H + \nu_e$ 1.442 MeV

100%

0.24%

${}^{2}H + p \rightarrow {}^{3}He + \gamma$

85%

15%

PPI

${}^{3}He + {}^{3}He \rightarrow {}^{4}He + 2p$

${}^{3}He + {}^{4}He \rightarrow {}^{7}Be + \gamma$

hep

${}^{3}He + p \rightarrow {}^{4}He + e^{+} + \nu_e$

< 18.8 MeV

90%

10%

0.02%

${}^{7}Be + e^{-} \rightarrow {}^{7}Li + \nu_e$ 0.862 MeV

${}^{7}Be + e^{-} \rightarrow {}^{7}Li^{*} + \nu_e$ 0.384 MeV

${}^{7}Be + p \rightarrow {}^{8}B + \gamma$

${}^{8}B \rightarrow {}^{8}Be^{*} + e^{+} + \nu_e$

< 15 MeV

PPII

${}^{7}Li + p \rightarrow {}^{4}He + {}^{4}He$

PPIII

${}^{8}Be^{*} \rightarrow {}^{4}He + {}^{4}He$

Georg Raffelt, Max-Planck-Institut für Physik, München, Germany

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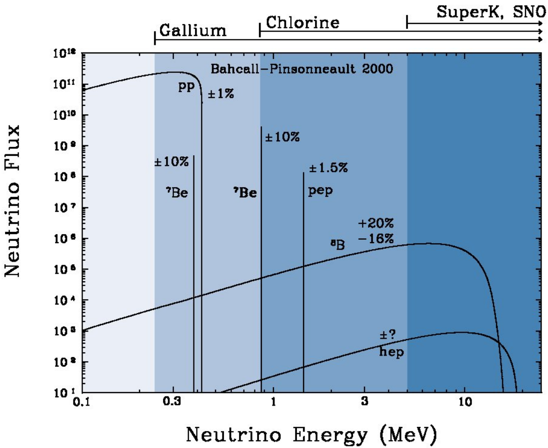
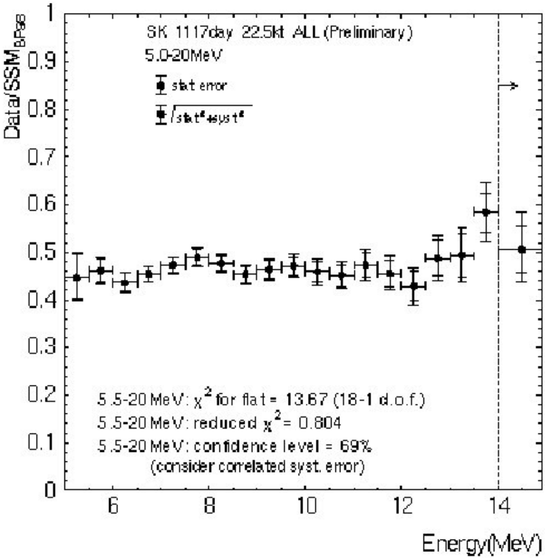
The solar neutrino spectra



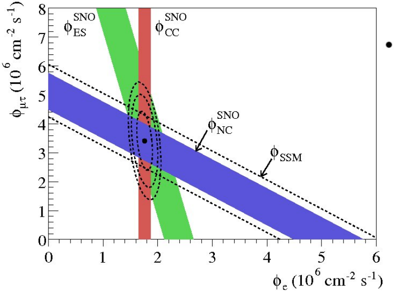
Mystery of missing solar neutrinos



Where did the missing neutrinos go ?

Super-Kamiokande

Solar ν_e convert to ν_μ and ν_τ



- $\nu_e\ D \to p\ p\ e^-$
- $\nu_{e,\mu,\tau}\ e^- \to \nu_{e,\mu,\tau}\ e^-$
- $\nu_{e,\mu,\tau}\ D \to n\ p\ \nu_{e,\mu,\tau}$

SNO

- ν_e oscillate into ν_μ and ν_τ

2-ν level crossing: MSW resonance

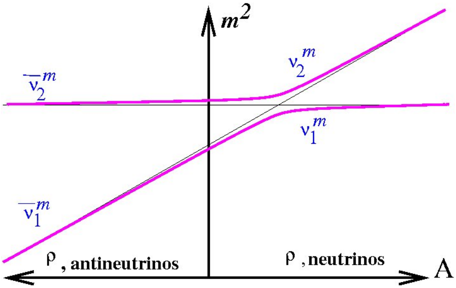
- Effective Hamiltonian:

$$H = \frac{1}{4E}\begin{pmatrix} -\Delta m^2 \cos 2\theta + 2A & \Delta m^2 \sin 2\theta \\ \Delta m^2 \sin 2\theta & \Delta m^2 \cos 2\theta \end{pmatrix}$$

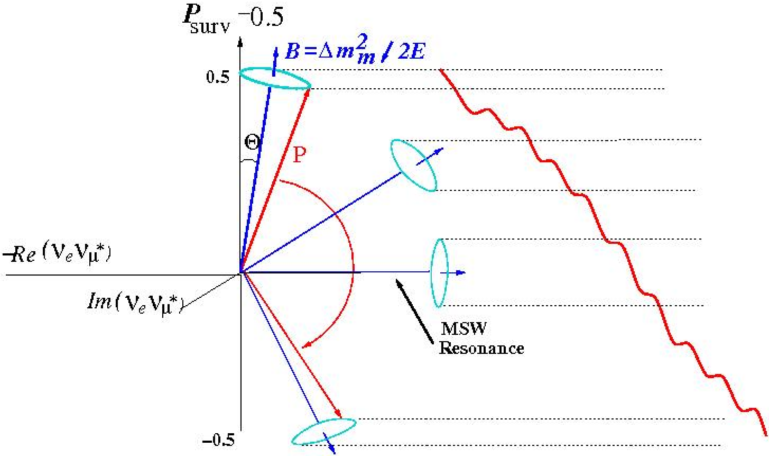
$$(A = 2\sqrt{2} G_F N_e E)$$

- Eigenvalues:

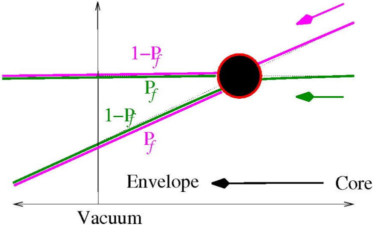
$$\frac{m_i^2}{2E} = \frac{1}{2E}\left[\frac{A}{2} \mp \sqrt{(\Delta m^2 \cos 2\theta - A)^2 + (\Delta m^2 \sin 2\theta)^2}\right]$$



Precession picture of MSW resonance



Adiabaticity at a resonance



$$P_f \approx exp(-\pi\gamma/2)$$

Landau'1932, Zener'1932

$$\gamma \equiv \frac{\Delta m^2}{2E}\frac{\sin^2 2\theta}{\cos 2\theta}\left(\frac{1}{n_e}\frac{dn_e}{dr}\right)^{-1}$$

$$\gamma \gg 1 \Rightarrow P_f \ll 1 \Rightarrow$$

Adiabatic resonance

P_f depends on: Δm^2, mixing angle $\theta_\odot$, density profile

Solution of the solar neutrino puzzle

- Survival probability:

$$P(\nu_e \to \nu_e) \approx P_f \cos^2 \theta_\odot + (1 - P_f) \sin^2 \theta_\odot$$

- No oscillations ! (Mass eigenstates have decohered)

$$\Delta m^2_\odot \approx (7.2\text{–}9.5) \times 10^{-5}\ \mathrm{eV}^2$$

Mixing angle $\theta_\odot \approx 28^\circ\text{–}36^\circ$

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Our current knowledge about neutrinos

- ν_e, ν_μ, ν_τ mix among each other
- Atmospheric neutrinos: $\Delta m^2_{\rm atm} \approx 2 \times 10^{-3}$ eV^2, $\theta_{\rm atm} \approx 45^\circ$
- Solar neutrinos: $\Delta m^2_\odot \approx 8 \times 10^{-5}$ eV^2, $\theta_\odot \approx 32^\circ$
- Reactor neutrinos: the “third” angle: very small ($\theta_{13} < 12^\circ$, may even be zero).

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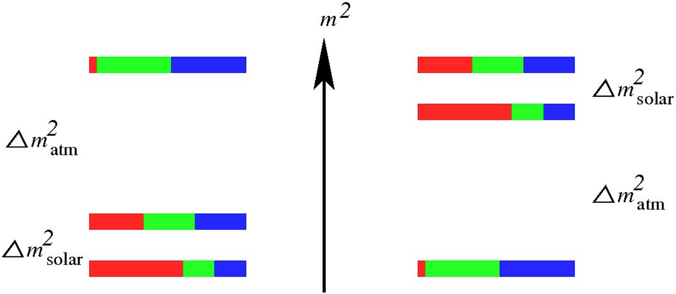
Open questions in neutrino physics

- Mass hierarchy: Normal or Inverted ?
  (red ν_e, green ν_μ, blue ν_τ)



- Absolute neutrino masses
- Are there more than 3 neutrinos ?
- CP violation ? own antiparticles ? ...

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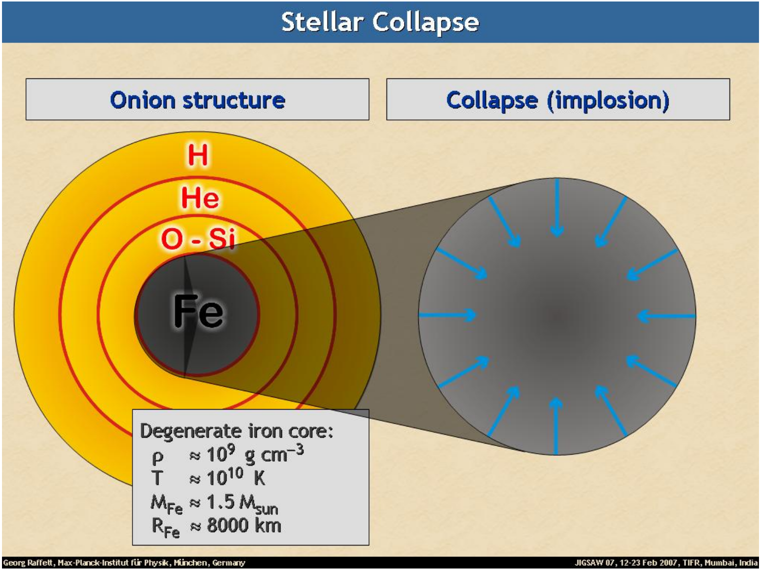
The onion ring structure



Stellar Collapse

Onion structure

Collapse (implosion)

Degenerate iron core:

$\rho \approx 10^9$ g cm^{-3}

$T \approx 10^{10}$ K

$M_{Fe} \approx 1.5\, M_{sun}$

$R_{Fe} \approx 8000$ km

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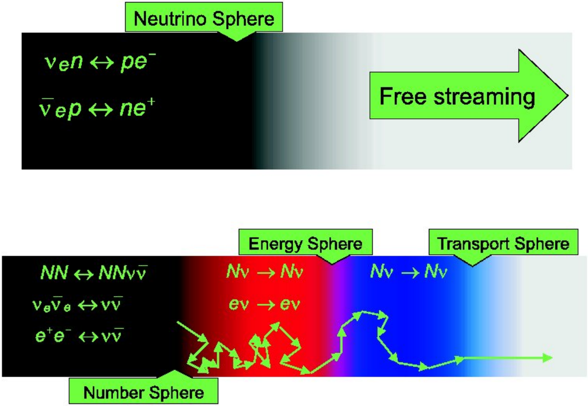
Trapped neutrinos before the collapse

- Neutrinos trapped inside “neutrinospheres” around $\rho \sim 10^{10}$g/cc.



- Escaping neutrinos: $\langle E_{\nu_e} \rangle < \langle E_{\bar{\nu}_e} \rangle < \langle E_{\nu_x} \rangle$

Core collapse, shock wave, and explosion

Gravitational core collapse $\Rightarrow$ Shock Wave

Neutronization burst:

ν_e emitted for $\sim$ 10 ms

Cooling through neutrino emission: $\nu_e, \bar{\nu}_e, \nu_\mu, \bar{\nu}_\mu, \nu_\tau, \bar{\nu}_\tau$

Duration: About 10 sec
Emission of most of the SN energy in neutrinos

¿¿¿ *Explosion* ???

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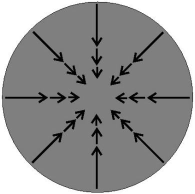
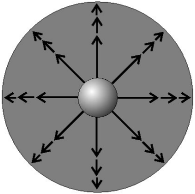
Gravitational core collapse ⇒ Shock Wave





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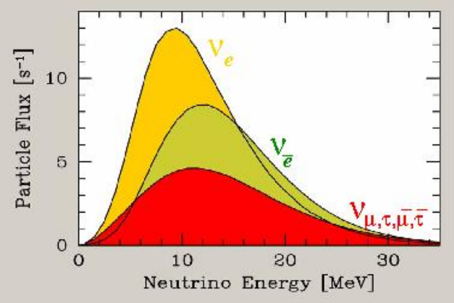
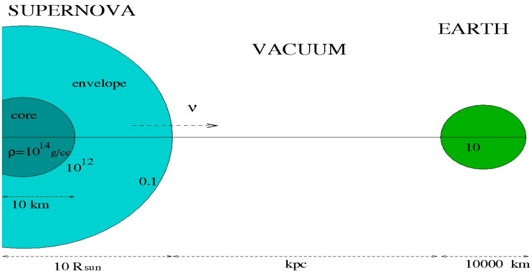
¿¿¿ *Explosion* ???

Primary fluxes and spectra



- Almost blackbody spectra, slightly “pinched”
- Energy hierarchy: $E_0(\nu_e) < E_0(\bar{\nu}_e) < E_0(\nu_x)$
- $E_0(\nu_e) \approx$ 10–12 MeV
  $E_0(\bar{\nu}_e) \approx$ 13–16 MeV
  $E_0(\nu_x) \approx$ 15–25 MeV

Propagation through matter of varying density



Inside the SN: ***flavour conversion***

Collective effects and MSW matter effects

Between the SN and Earth: ***no flavour conversion***

Mass eigenstates travel independently

Inside the Earth: ***flavour conversion***

MSW matter effects (*if detector is on the other side*)

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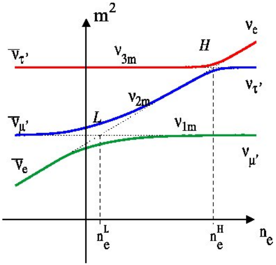
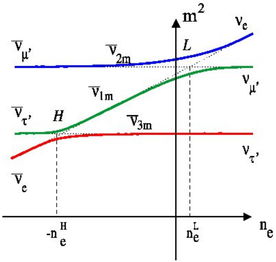
MSW Resonances inside a SN

Normal mass ordering



Inverted mass ordering



H resonance: $(\Delta m^2_{\rm atm}, \theta_{13})$, $\rho \sim 10^3$–10^4 g/cc

- In $\nu(\bar{\nu})$ for normal (inverted) hierarchy
- Adiabatic (non-adiabatic) for $\sin^2\theta_{13} \gtrsim 10^{-3} (\lesssim 10^{-5})$

L resonance: $(\Delta m^2_\odot, \theta_\odot)$, $\rho \sim 10$–100 g/cc

- Always adiabatic, always in ν

Mixing of fluxes due to MSW resonances

Mixture of initial fluxes:

$$\begin{aligned} F_{\nu_e} &= p\, F^0_{\nu_e} + (1-p)\, F^0_{\nu_x} \; , \\ F_{\bar{\nu}_e} &= \bar{p}\, F^0_{\bar{\nu}_e} + (1-\bar{p})\, F^0_{\nu_x} \end{aligned} \tag{1}$$

Survival probabilities in different scenarios:

	Hierarchy	$\sin^2\theta_{13}$	p	$\bar{p}$
A	Normal	Large	0	$\cos^2\theta_\odot$
B	Inverted	Large	$\sin^2\theta_\odot$	0
C	Normal	Small	$\sin^2\theta_\odot$	$\cos^2\theta_\odot$
D	Inverted	Small	$\sin^2\theta_\odot$	$\cos^2\theta_\odot$

- “Small”: $\sin^2\theta_{13} \lesssim 10^{-5}$, “Large”: $\sin^2\theta_{13} \gtrsim 10^{-3}$.
- Scenarios C and D are degenerate !!

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D	Inverted	Small	$\sin^2\theta_\odot$	$\cos^2\theta_\odot$

- “Small”: $\sin^2\theta_{13} \lesssim 10^{-5}$, “Large”: $\sin^2\theta_{13} \gtrsim 10^{-3}$.
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Lessons from Lecture 1

- Matter affects neutrino mixing and flavour conversions
- MSW resonances sensitive to mass hierarchy
- $\sin^2\theta_{13} \gtrsim 10^{-3}$ and $\sin^2\theta_{13} \lesssim 10^{-5}$ give distinct results

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ν_e emitted for $\sim$ 10 ms

Cooling through neutrino emission: $\nu_e, \bar{\nu}_e, \nu_\mu, \bar{\nu}_\mu, \nu_\tau, \bar{\nu}_\tau$

Duration: About 10 sec
Emission of most of the SN energy in neutrinos

¿¿¿ *Explosion* ???

Core collapse, shock wave, and explosion

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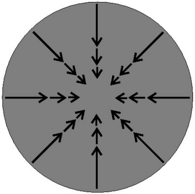

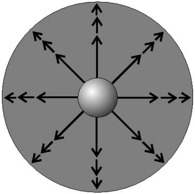

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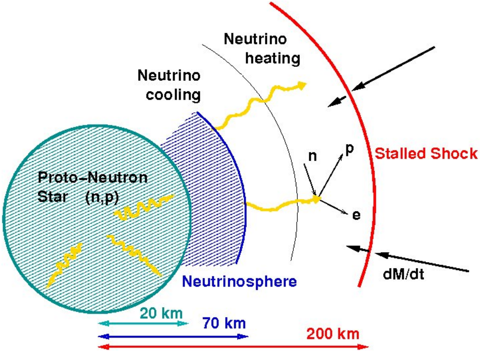
¿¿¿ *Explosion* ???

Role of neutrinos in explosion

Neutrino heating needed for pushing the shock wave

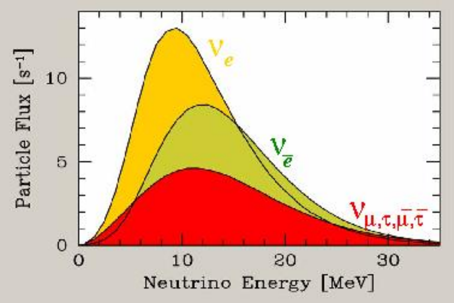


- Neutrino heating essential, but not enough
- No spherically symmetric (1-D) simulations show robust explosions
- Large scale convections required for explosion

An explosion

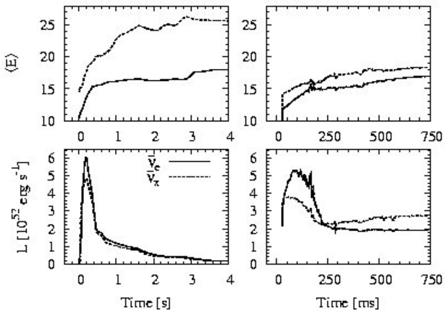
The explosion movie

Primary fluxes and spectra



- Almost blackbody spectra, slightly “pinched”
- Energy hierarchy: $E_0(\nu_e) < E_0(\bar{\nu}_e) < E_0(\nu_x)$
- $E_0(\nu_e) \approx$ 10–12 MeV
  $E_0(\bar{\nu}_e) \approx$ 13–16 MeV
  $E_0(\nu_x) \approx$ 15–25 MeV

Flavor-dependence of neutrino fluxes

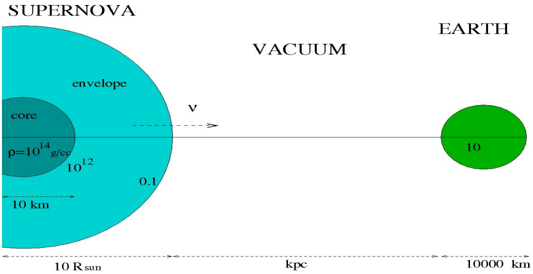


solid line: $\bar{\nu}_e$
dotted line: $\bar{\nu}_x$

Model	$\langle E_0(\nu_e)\rangle$	$\langle E_0(\bar{\nu}_e)\rangle$	$\langle E_0(\nu_x)\rangle$	$\frac{\Phi_0(\nu_e)}{\Phi_0(\nu_x)}$	$\frac{\Phi_0(\bar{\nu}_e)}{\Phi_0(\nu_x)}$
Garching (G)	12	15	18	0.8	0.8
Livermore (L)	12	15	24	2.0	1.6

G. G. Raffelt, M. T. Keil, R. Buras, H. T. Janka and M. Rampp, astro-ph/0303226
T. Totani, K. Sato, H. E. Dalhed and J. R. Wilson, Astrophys. J. 496, 216 (1998)

Propagation through matter of varying density



Inside the SN: *flavour conversion*

Collective effects and MSW matter effects

Between the SN and Earth: *no flavour conversion*

Mass eigenstates travel independently

Inside the Earth: *flavour conversion*

MSW matter effects (*if detector is on the other side*)

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Nonlinear effects due to $\nu - \nu$ coherent interactions

- Large neutrino density ⇒ substantial ν–ν potential

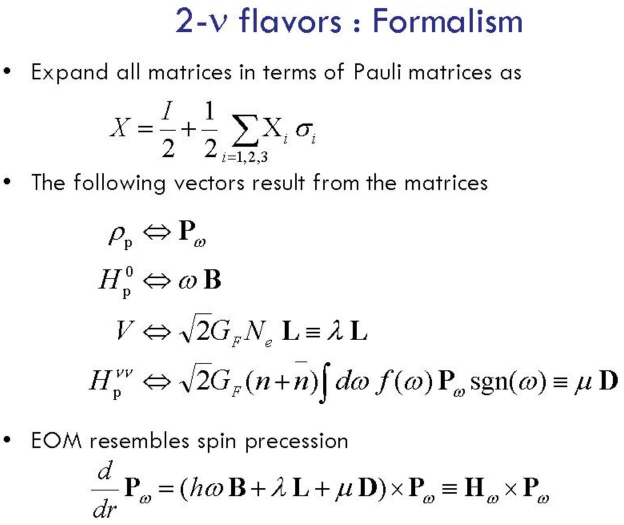
$$H = H_{vac} + H_{MSW} + H_{\nu\nu}$$

$$\begin{aligned} H_{vac}(\vec{p}) &= M^2/(2p) \\ H_{MSW} &= \sqrt{2}G_F n_{e^-} diag(1,0,0) \\ H_{\nu\nu}(\vec{p}) &= \sqrt{2}G_F \int \frac{d^3q}{(2\pi)^3}(1-\cos\theta_{pq})\left(\rho(\vec{q}) - \bar{\rho}(\vec{q})\right) \end{aligned}$$

- $$\frac{d\rho}{dt} = i\,[H(\rho), \rho] \quad \Rightarrow \qquad \text{Nonlinear effects !}$$

Synchronized osc. → Bipolar osc. → Spectral split

Polarization analogy in two flavours



2-ν flavors : Formalism

- Expand all matrices in terms of Pauli matrices as

$$X = \frac{I}{2} + \frac{1}{2}\sum_{i=1,2,3} \mathrm{X}_i\,\sigma_i$$

- The following vectors result from the matrices

$$\rho_{\mathrm{p}} \Leftrightarrow \mathbf{P}_\omega$$

$$H^{0}_{\mathrm{p}} \Leftrightarrow \omega\,\mathbf{B}$$

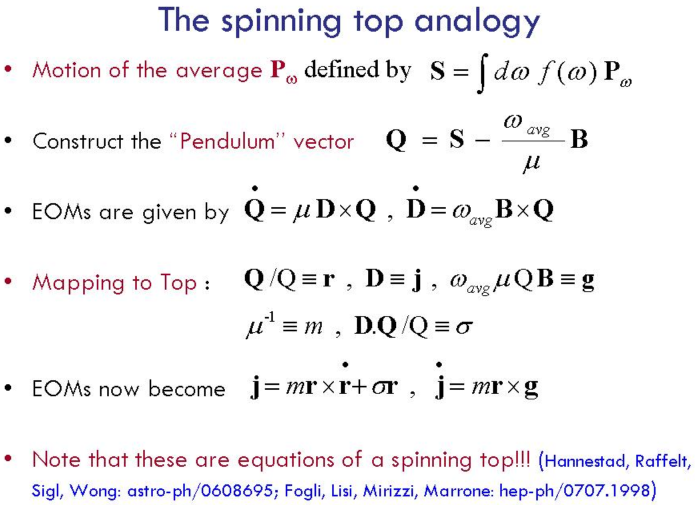
$$V \Leftrightarrow \sqrt{2}G_F N_e\,\mathbf{L} \equiv \lambda\,\mathbf{L}$$

$$H^{\nu\nu}_{\mathrm{p}} \Leftrightarrow \sqrt{2}G_F(n+\bar{n})\int d\omega\, f(\omega)\,\mathbf{P}_\omega\,\mathrm{sgn}(\omega) \equiv \mu\,\mathbf{D}$$

- EOM resembles spin precession

$$\frac{d}{dr}\mathbf{P}_\omega = (h\omega\,\mathbf{B} + \lambda\,\mathbf{L} + \mu\,\mathbf{D}) \times \mathbf{P}_\omega \equiv \mathbf{H}_\omega \times \mathbf{P}_\omega$$

Analogy to a spinning top

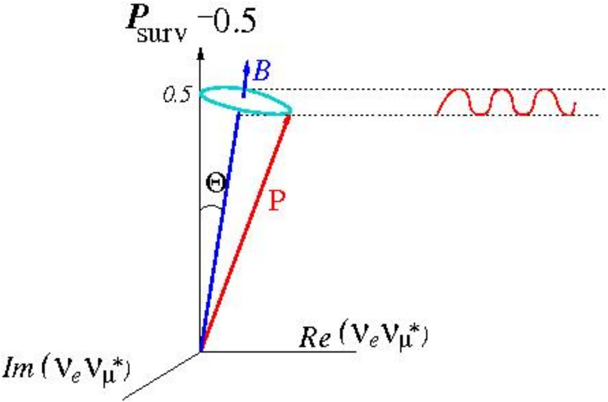


The spinning top analogy

- Motion of the average $\mathbf{P}_\omega$ defined by $\mathbf{S} = \int d\omega\, f(\omega)\, \mathbf{P}_\omega$
- Construct the "Pendulum" vector $\mathbf{Q} = \mathbf{S} - \frac{\omega_{avg}}{\mu}\mathbf{B}$
- EOMs are given by $\dot{\mathbf{Q}} = \mu\,\mathbf{D}\times\mathbf{Q}$, $\dot{\mathbf{D}} = \omega_{avg}\mathbf{B}\times\mathbf{Q}$
- Mapping to Top : $\mathbf{Q}/\mathrm{Q} \equiv \mathbf{r}$, $\mathbf{D} \equiv \mathbf{j}$, $\omega_{avg}\mu\mathrm{Q}\mathbf{B} \equiv \mathbf{g}$

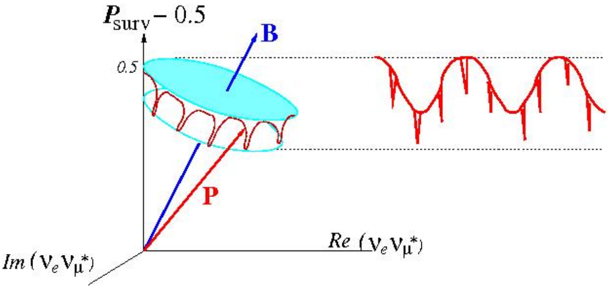
  $\mu^{-1} \equiv m$, $\mathbf{D.Q}/\mathrm{Q} \equiv \sigma$
- EOMs now become $\mathbf{j} = m\mathbf{r}\times\dot{\mathbf{r}} + \sigma\mathbf{r}$, $\dot{\mathbf{j}} = m\mathbf{r}\times\mathbf{g}$
- Note that these are equations of a spinning top!!! (Hannestad, Raffelt, Sigl, Wong: astro-ph/0608695; Fogli, Lisi, Mirizzi, Marrone: hep-ph/0707.1998)

Synchronized oscillations



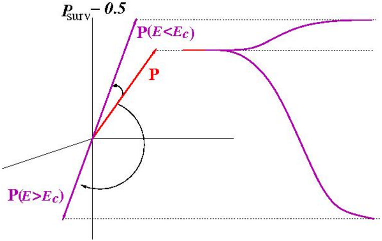
- ν and $\bar{\nu}$ of all energies oscillate with the same frequency
- No significant flavour change since mixing angle is small

Bipolar oscillations

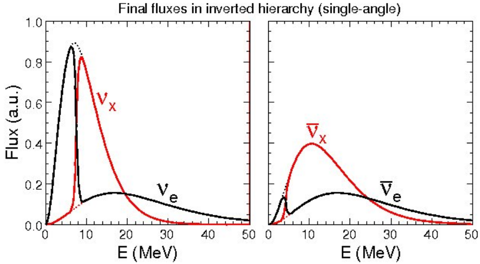


- Coherent $\nu_e \bar{\nu}_e \leftrightarrow \nu_x \bar{\nu}_x$ oscillations
- A nutating top ??
- Take place in inverted hierarchy
- Even $\theta_{13} \lesssim 10^{-10}$ OK !
- Prepare neutrinos for the “spectral split”

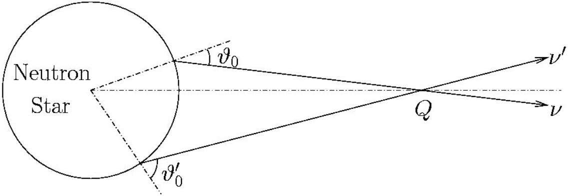
Spectral split



- $\bar{\nu}_e$ and $\bar{\nu}_x$ spectra interchange completely
- ν_e and ν_x spectra interchange for $E > E_c$
- Occurs in inverted hierarchy



A caveat: multi-angle effects



H. Duan, G. Fuller, J. Carlson, Y. Qian,PRL 97, 241101 (2006)

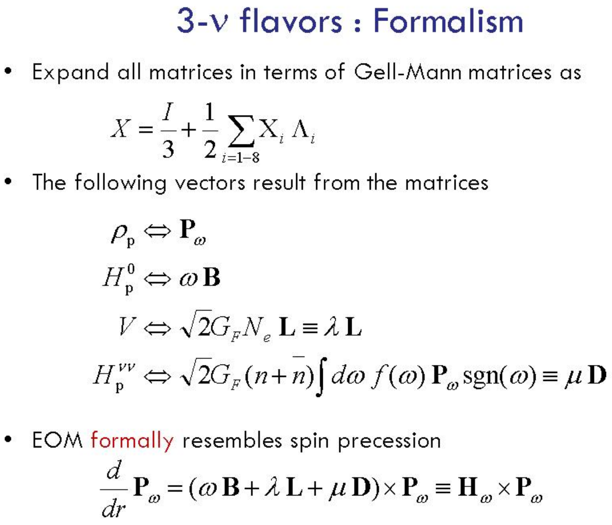
- "Multi-angle decoherence" during collective oscillations suppressed by ν–$\bar{\nu}$ asymmetry

  A.Esteban-Pretel et al.,PRD76, 125018 (2007)

- "Single-angle" evolution along lines of neutrino flux works even for non-spherical geometries, as long as coherence is maintained

  B.Dasgupta et al., arXiv:0805.3300

A peek into three-flavour analysis



3-ν flavors : Formalism

- Expand all matrices in terms of Gell-Mann matrices as

$$X = \frac{I}{3} + \frac{1}{2} \sum_{i=1-8} \mathrm{X}_i \, \Lambda_i$$

- The following vectors result from the matrices

$$\rho_{\mathrm{p}} \Leftrightarrow \mathbf{P}_\omega$$

$$H_{\mathrm{p}}^0 \Leftrightarrow \omega \, \mathbf{B}$$

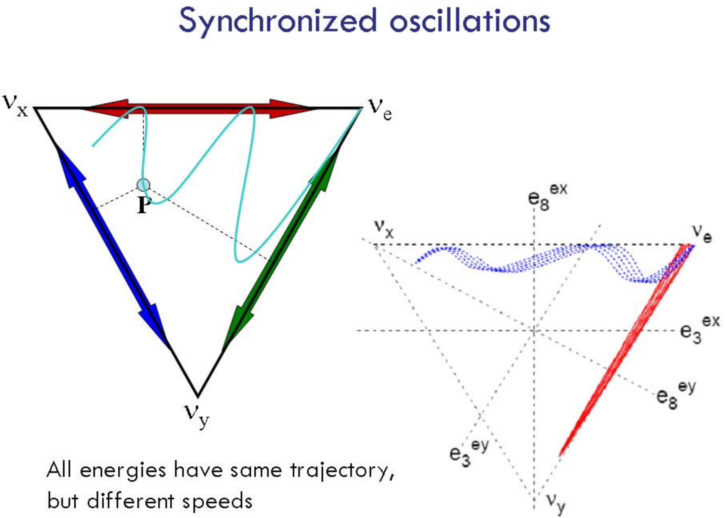
$$V \Leftrightarrow \sqrt{2} G_F N_e \, \mathbf{L} \equiv \lambda \, \mathbf{L}$$

$$H_{\mathrm{p}}^{\nu\nu} \Leftrightarrow \sqrt{2} G_F (n + \bar{n}) \int d\omega \, f(\omega) \, \mathbf{P}_\omega \, \mathrm{sgn}(\omega) \equiv \mu \, \mathbf{D}$$

- EOM formally resembles spin precession

$$\frac{d}{dr} \mathbf{P}_\omega = (\omega \, \mathbf{B} + \lambda \, \mathbf{L} + \mu \, \mathbf{D}) \times \mathbf{P}_\omega \equiv \mathbf{H}_\omega \times \mathbf{P}_\omega$$

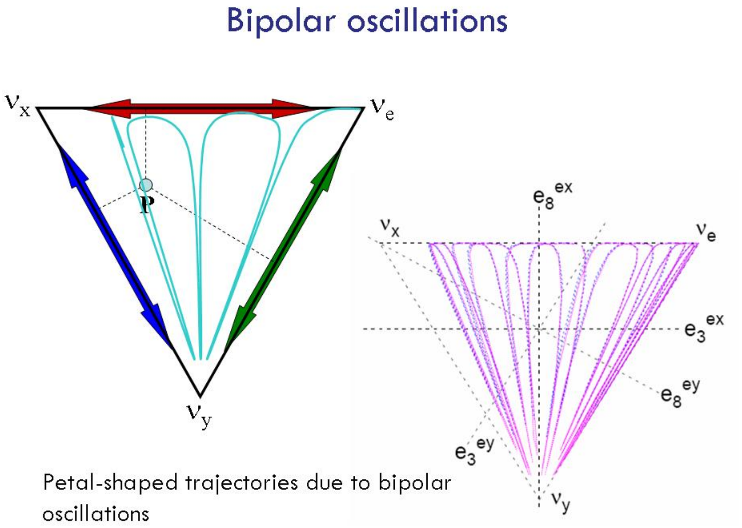
Synchronized oscillations: 3 flavours



Synchronized oscillations

All energies have same trajectory,
but different speeds

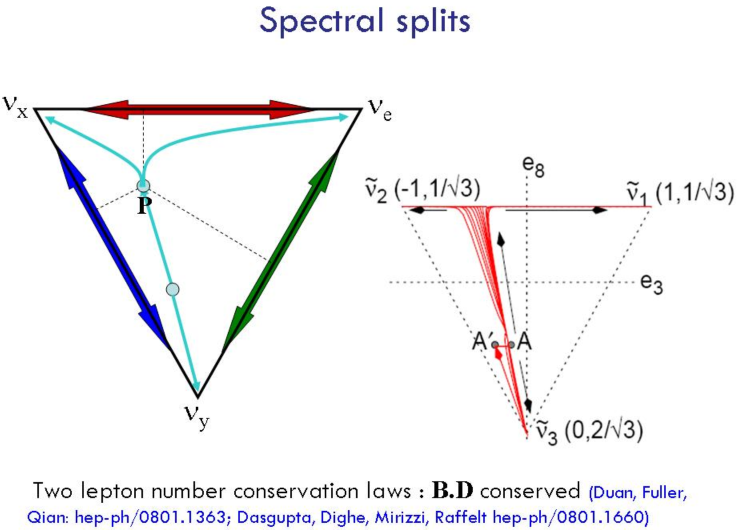
Bipolar oscillations: 3 flavours



Bipolar oscillations

Petal-shaped trajectories due to bipolar oscillations

Spectral split: 3 flavours



Spectral splits

Two lepton number conservation laws : **B.D** conserved (Duan, Fuller, Qian: hep-ph/0801.1363; Dasgupta, Dighe, Mirizzi, Raffelt hep-ph/0801.1660)

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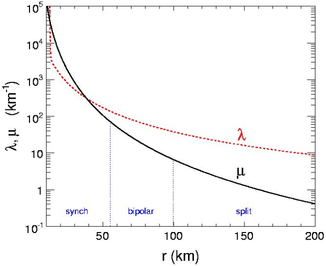
Sequential dominance of processes



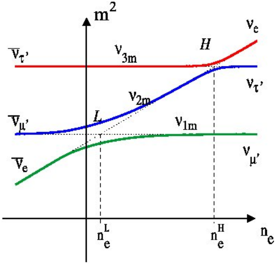
- $\mu \equiv \sqrt{2} G_F (N_\nu + N_{\bar{\nu}})$
- $\lambda \equiv \sqrt{2} G_F N_e$

- $r \lesssim 200$ km: collective effects dominate
- $r \gtrsim 200$ km: standard MSW matter effects dominate

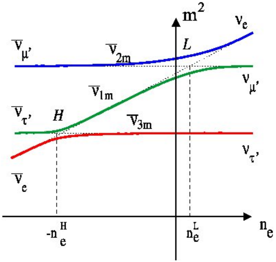
G.L.Fogli, E. Lisi, A. Marrone, A. Mirizzi, JCAP 0712, 010 (2007)

MSW Resonances inside a SN

Normal mass ordering



Inverted mass ordering



H resonance: $(\Delta m^2_{\rm atm}, \theta_{13})$, $\rho \sim 10^3$–10^4 g/cc

- In $\nu(\bar\nu)$ for normal (inverted) hierarchy
- Adiabatic (non-adiabatic) for $\sin^2\theta_{13} \gtrsim 10^{-3} (\lesssim 10^{-5})$

L resonance: $(\Delta m^2_\odot, \theta_\odot)$, $\rho \sim 10$–100 g/cc

- Always adiabatic, always in ν

Fluxes arriving at the Earth

Mixture of initial fluxes:

$$\begin{aligned} F_{\nu_e} &= p\, F^0_{\nu_e} + (1-p)\, F^0_{\nu_x}\,, \\ F_{\bar\nu_e} &= \bar p\, F^0_{\bar\nu_e} + (1-\bar p)\, F^0_{\nu_x} \end{aligned} \tag{2}$$

Survival probabilities in different scenarios:

	Hierarchy	$\sin^2\theta_{13}$	p	$\bar p$
A	Normal	Large	0	$\cos^2\theta_\odot$
B	Inverted	Large	$\cos^2\theta_\odot \mid 0$	$\cos^2\theta_\odot$
C	Normal	Small	$\sin^2\theta_\odot$	$\cos^2\theta_\odot$
D	Inverted	Small	$\cos^2\theta_\odot \mid 0$	0

- "Small": $\sin^2\theta_{13} \lesssim 10^{-5}$, "Large": $\sin^2\theta_{13} \gtrsim 10^{-3}$.
- All four scenarios separable in principle !!

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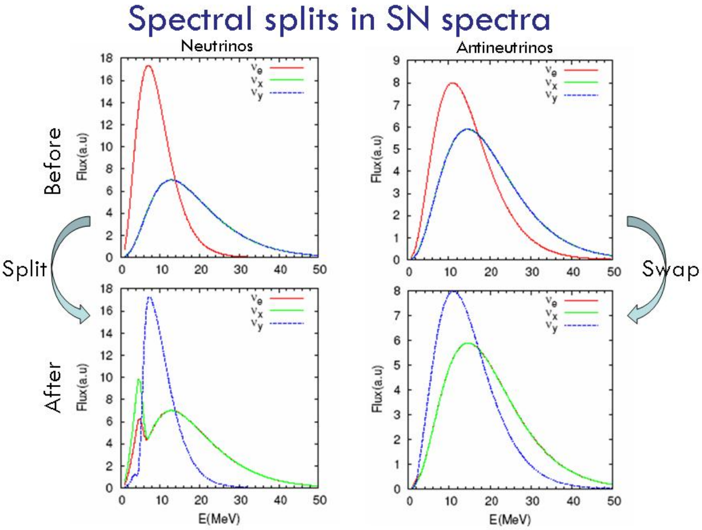
Final spectra for inverted hierarchy ($F^0_{\nu_e} > F^0_{\bar{\nu}_e} > F^0_{\nu_x}$)



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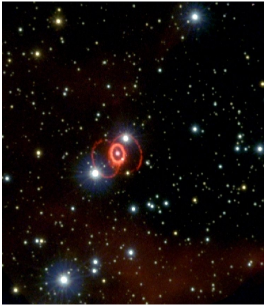
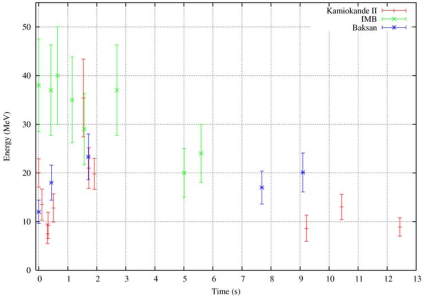
SN1987A



(Hubble image)



- Confirmed the SN cooling mechanism through neutrinos
- Number of events too small to say anything concrete about neutrino mixing
- Some constraints on SN parameters obtained

Signal expected from a galactic SN (10 kpc)

Water Cherenkov detector / IceCUBE:

- $\bar{\nu}_e p \rightarrow n e^+$: $\approx$ 7000 – 12000*
- $\nu e^- \rightarrow \nu e^-$: $\approx$ 200 – 300*
- $\nu_e +^{16} O \rightarrow X + e^-$: $\approx$ 150–800*

* Events expected at Super-Kamiokande with a galactic SN at 10 kpc

Carbon-based scintillation detector:

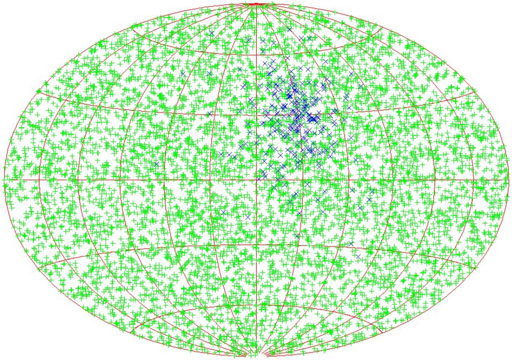
- $\bar{\nu}_e p \rightarrow n e^+$
- $\nu + {}^{12}C \rightarrow \nu + X + \gamma$ (15.11 MeV)

Liquid Argon detector:

- $\nu_e + {}^{40}Ar \rightarrow {}^{40}K^* + e^-$

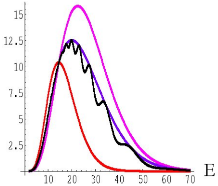
Pointing to the SN in advance

- Neutrinos reach 6-24 hours before the light from SN explosion (SNEWS network)
- $\bar{\nu}_e p \to n e^+$: nearly isotropic background
- $\nu e^- \to \nu e^-$: forward-peaked “signal”
- Background-to-signal ratio: $N_B/N_S \approx 30\text{–}50$
- SN at 10 kpc may be detected within a cone of $\sim 5^\circ$ at SK



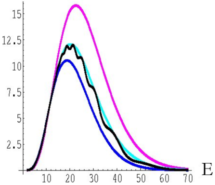
Earth matter effects

Neutrinos



(ν_e, ν_x, mixed ν)

Antineutrinos



($\bar{\nu}_e$, $\bar{\nu}_x$, mixed $\bar{\nu}$)

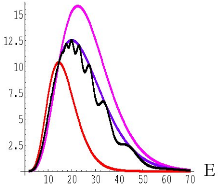
- “Earth effect” oscillations

Presence or absence of Earth matter effects:

	Hierarchy	$\sin^2\theta_{13}$	ν_e	$\bar{\nu}_e$
A	Normal	Large	X	$\surd$
B	Inverted	Large	X	$\surd$
C	Normal	Small	$\surd$	$\surd$
D	Inverted	Small	X	X

Earth matter effects

Neutrinos



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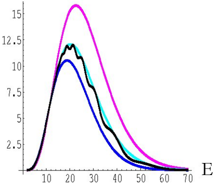


($\bar{\nu}_e$, $\bar{\nu}_x$, mixed $\bar{\nu}$)

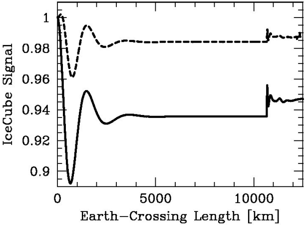
- “Earth effect” oscillations

Presence or absence of Earth matter effects:

	Hierarchy	$\sin^2\theta_{13}$	ν_e	$\bar{\nu}_e$
A	Normal	Large	X	√
B	Inverted	Large	X	√
C	Normal	Small	√	√
D	Inverted	Small	X	X

IceCube as a co-detector with HK

- Total Cherenkov count in IceCube increases beyond statistical backround fluctuations during a SN burst
  F.Halzen, J.Jacobsen, E.Zas, PRD53, 7359 (1996)
- This signal can be determined to a statistical accuracy of $\sim 0.25\%$ for a SN at 10 kpc.
- The extent of Earth effects changes by 3–4 % between the accretion phase (first 0.5 sec) and the cooling phase.
- Absolute calibration not essential



AD, M. Keil, G. Raffelt,
JCAP 0306:005 (2003)

Collective effects will change the ratio

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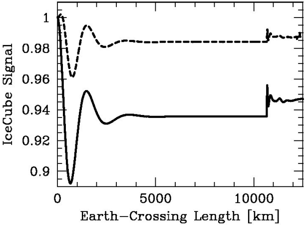


AD, M. Keil, G. Raffelt,
JCAP 0306:005 (2003)

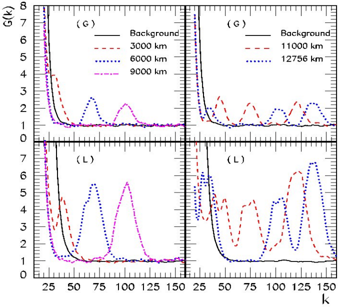
Collective effects will change the ratio

Earth effects through Fourier Transform

Power spectrum: $G_N(k) = \frac{1}{N} \left| \sum_{events} e^{iky} \right|^2$

$(y \equiv 25\ MeV/E)$

- Model independence of peak positions at a scintillator:



AD, M. Kachelrieß, G. Raffelt, R. Tomàs, JCAP 0401:004 (2004)

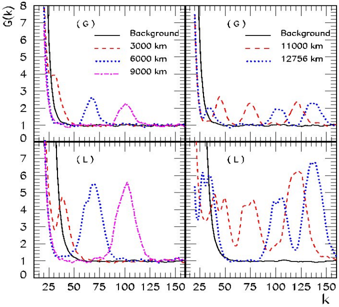
Collective effects will not change peak positions

Earth effects through Fourier Transform

Power spectrum: $G_N(k) = \frac{1}{N} \left|\sum_{events} e^{iky}\right|^2$

$(y \equiv 25\ MeV/E)$

- Model independence of peak positions at a scintillator:

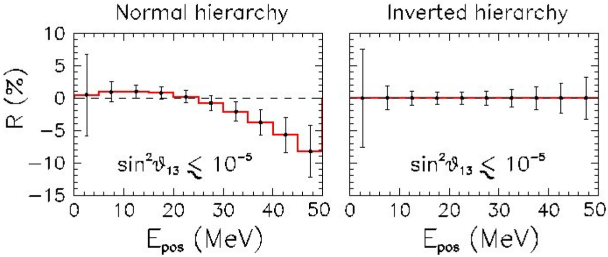


AD, M. Kachelrieß, G. Raffelt,
R. Tomàs, JCAP 0401:004 (2004)

Collective effects will not change peak positions

Earth matter effects from two Water Cherenkovs

$$R \equiv \frac{N(shadowed) - N(unshadowed)}{N(unshadowed)}$$



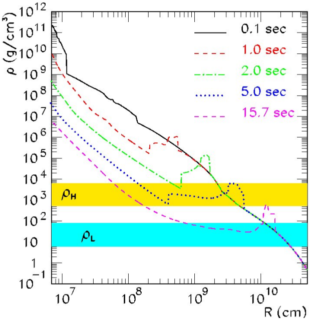
Robust experimental signature, thanks to Collective Effects

- Earth effects can distinguish hierarchies even for $\theta_{13} \rightarrow 0$

B.Dasgupta et al., arXiv:0802.1481

Shock wave and adiabaticity breaking

When shock wave passes through a resonance region (density ρ_H or ρ_L):



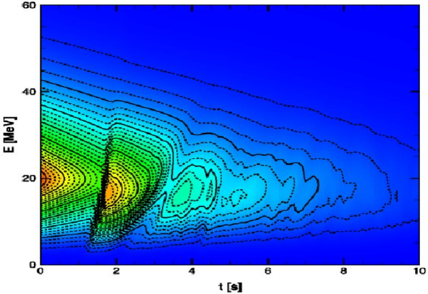
- adiabatic resonances may become momentarily non-adiabatic
  scenario A → scenario C
  scenario B → scenario D
- Sharp changes in the final spectra even if the primary spectra change smoothly

R. C. Schirato, G. M. Fuller, astro-ph/0205390

G. L. Fogli, E. Lisi, D. Montanino and A. Mirizzi, PRD 68, 033005 (2003)

Shock wave effects

- Time dependent spectral evolution



Kneller, Mclaughlin, Brockman, PRD77, 045023 (2008)

Presence or absence of shock effects

	Hierarchy	$\sin^2\theta_{13}$	ν_e	$\bar{\nu}_e$
A	Normal	Large	$\surd$	$\surd$
B	Inverted	Large	X	$\surd$
C	Normal	Small	X	X
D	Inverted	Small	X	X

Shock wave effects

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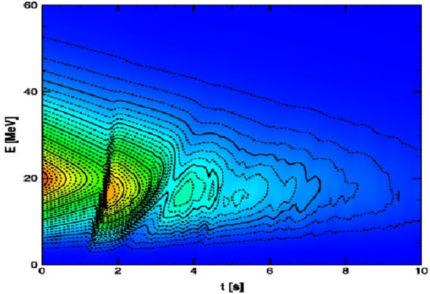

Kneller, Mclaughlin,
Brockman,
PRD77, 045023 (2008)

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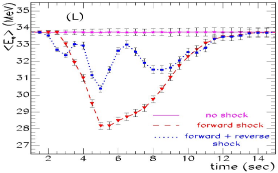
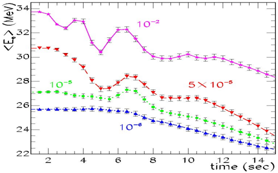
C	Normal	Small	X	X
D	Inverted	Small	X	X

Shock wave effect on survival probabaility

The shock wave movie

Double/single dip at a megaton water Cherenkov





$$\left.\begin{array}{l}\text{Single (Double) dip in } \langle E_e \rangle \\ \text{Single (Double) peak in } \langle E_e^2 \rangle / \langle E_e \rangle^2\end{array}\right\} \text{ for Forward (+ Reverse) shock}$$

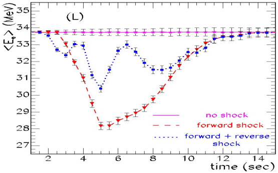
Double/single dip

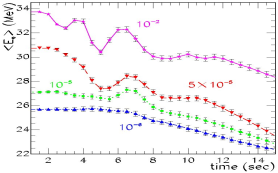
- robust under monotonically decreasing average energy
- In ν_e ($\bar{\nu}_e$) for normal (inverted) hierarchy for $\sin^2\theta_{13} \gtrsim 10^{-5}$

R.Tomas et al., JCAP **0409**, 015 (2004)

Collective effects ⇒ dip ↔ peak

Double/single dip at a megaton water Cherenkov





Single (Double) dip in $\langle E_e \rangle$
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$\Big\}$ for Forward (+ Reverse) shock

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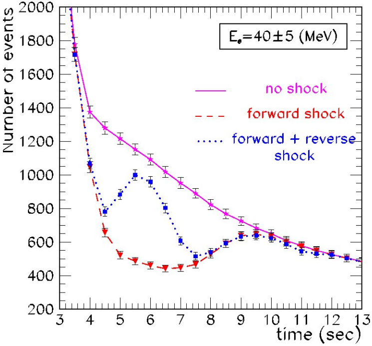
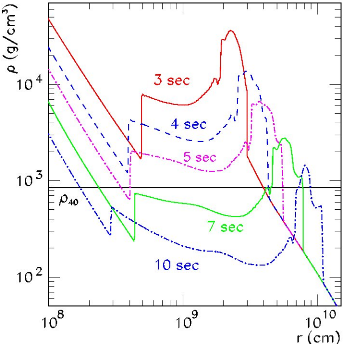
R.Tomas et al., JCAP **0409**, 015 (2004)

Collective effects $\Rightarrow$ dip $\leftrightarrow$ peak

Tracking the shock fronts





- At $t \approx 4.5$ sec, (reverse) shock at ρ_{40}
- At $t \approx 7.5$ sec, (forward) shock at ρ_{40}
- Multiple energy bins $\Rightarrow$ the times the shock fronts reach different densities of $\rho \sim 10^2$–10^4 g/cc

Future perspective

Theoretical challenges

- Neutrino transport inside the SN, primary spectra
- Aspects of the nonlinear effects: spectral split, multi-angle effects, decoherence, turbulent effects

Experimental challenges

- Reconstruction of ν_e spectrum (liq Ar detector ?)
- Multiple megaton-class water Cherenkov detectors

Expected bonanza

- Neutrino mass hierarchy
- Upper/lower bounds on θ_{13}
- Understanding of shock wave propagation

See Neutrino2008 Proceedings, arXiv:0809.2977

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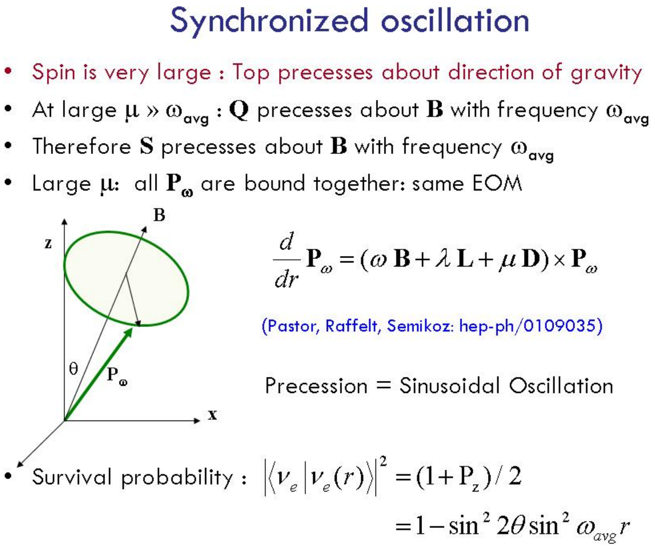
Synchronized oscillations



Synchronized oscillation

- Spin is very large : Top precesses about direction of gravity
- At large $\mu \gg \omega_{\mathrm{avg}}$: $\mathbf{Q}$ precesses about $\mathbf{B}$ with frequency ω_{avg}
- Therefore $\mathbf{S}$ precesses about $\mathbf{B}$ with frequency ω_{avg}
- Large μ: all $\mathbf{P}_\omega$ are bound together: same EOM

$$\frac{d}{dr}\mathbf{P}_\omega = (\omega\,\mathbf{B} + \lambda\,\mathbf{L} + \mu\,\mathbf{D}) \times \mathbf{P}_\omega$$

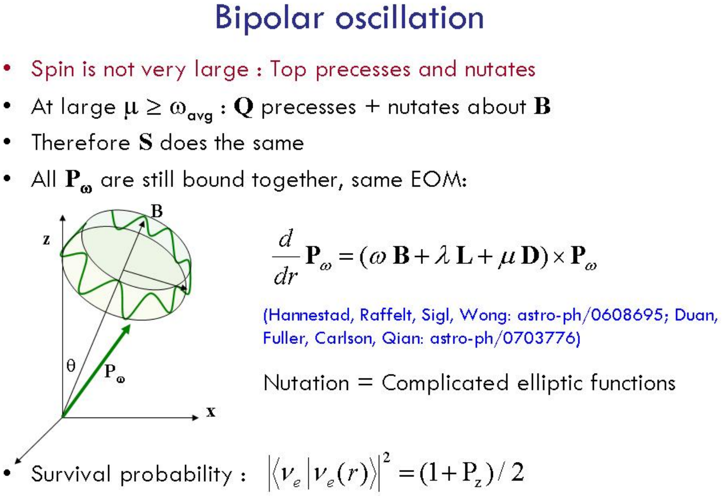
(Pastor, Raffelt, Semikoz: hep-ph/0109035)

Precession = Sinusoidal Oscillation

- Survival probability : $\left|\langle \nu_e | \nu_e(r) \rangle\right|^2 = (1 + \mathrm{P}_z)/2$

$$= 1 - \sin^2 2\theta \sin^2 \omega_{avg} r$$

Bipolar oscillations



Bipolar oscillation

- Spin is not very large : Top precesses and nutates
- At large $\mu \geq \omega_{\text{avg}}$: $\mathbf{Q}$ precesses + nutates about $\mathbf{B}$
- Therefore $\mathbf{S}$ does the same
- All $\mathbf{P}_\omega$ are still bound together, same EOM:

$$\frac{d}{dr}\mathbf{P}_\omega = (\omega\,\mathbf{B} + \lambda\,\mathbf{L} + \mu\,\mathbf{D}) \times \mathbf{P}_\omega$$

(Hannestad, Raffelt, Sigl, Wong: astro-ph/0608695; Duan, Fuller, Carlson, Qian: astro-ph/0703776)

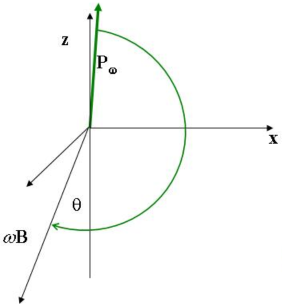
Nutation = Complicated elliptic functions

- Survival probability : $\left|\langle \nu_e | \nu_e(r) \rangle\right|^2 = (1 + \mathrm{P}_z)/2$

Spectral split

Adiabatic spectral split

- Top falls down when it slows down (when mass increases)
- If μ decreases slowly $\mathbf{P}_\omega$ keeps up with $\mathbf{H}_\omega$
- As $\mu\rightarrow 0$ from its large value : $\mathbf{P}_\omega$ aligns with $h\omega\mathbf{B}$
- For inverted hierarchy $\mathbf{P}_\omega$ has to flip, **BUT…**



$$\frac{d}{dr}\mathbf{B.D} \approx \mathbf{B}.\dot{\mathbf{D}} = \mathbf{B}.(\omega_{avg}\mathbf{B}\times\mathbf{Q}) = 0$$

- **B.D** is conserved so all P_ω can't flip
- Low energy modes anti-align
- All $\mathbf{P}_\omega$ with $\omega < \omega_c$ flip over
- Spectral Split

(Raffelt, Smirnov:hep-ph/0705.1830)