Investigating thermal abundance of semi-relativistic particles

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22 September 2007, Second UniverseNet School

Thermal freeze out

- Once upon a time in distant past....
  - $\Gamma \gtrsim H \Rightarrow A + B \leftrightarrow C + D$
  - $\Gamma < H \Rightarrow$ Freeze out
- Boltzmann Equation $\hat{L}[f] = C[f]$

$$\frac{dY}{dx} = \frac{-x\langle \sigma v \rangle s}{H(m)}(Y^2 - Y_{eq}^2)$$

where:- s = entropy

$Y = \frac{n}{s}$

$x = \frac{m}{T}$

- Relic density $\Omega h^2 = 2.8 \times 10^8 Y_\infty (m/GeV)$

Limiting cases

- Relativistic treatment ($m \ll T$)
  - $\Omega h^2 \propto m$, independent of $\langle \sigma v \rangle$
- Non-relativistic treatment ($m \gg T$)
  - Expansion of thermal average of cross-section in terms of velocity $\langle \sigma v \rangle = a + \frac{6b}{x}$
  - $Y_\infty = \frac{\sqrt{90}}{4\pi m M_{Pl} \sqrt{g_*(x_f)} \left(\frac{a}{x_f} + \frac{3b}{x_f^2} \right)} \propto \frac{1}{\langle \sigma v \rangle}$
  - $\Omega h^2 \propto m Y_\infty \propto \frac{1}{\langle \sigma v \rangle}$
- No known analytical solution in intermediate range ($m \simeq T$)

We need to...

- Modify the expression for abundance
  - Assuming Maxwell-Boltzmann distribution

$$Y_{eq} \equiv \frac{n_{eq}}{s} = 0.115 \frac{g}{g_{*s}} x^2 K_2(x)$$

$K_n(x)$ = Modified Bessel function

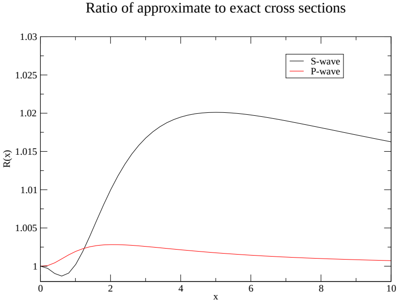
- New treatment for thermal averaging of cross-section

$$\langle \sigma v \rangle = \frac{1}{8m^4 T K_2^2(m/T)} \int_{4m^2}^{\infty} ds\, \sigma\, (s - 4m^2) \sqrt{s}\, K_1(\sqrt{s}/T)$$

Note :- s here is the Mandelstam variable.

Treating the thermal average

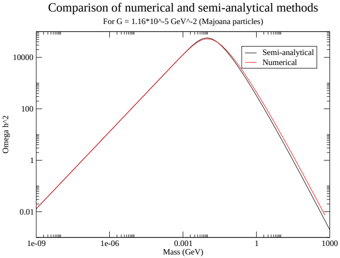
- Scenario :- Stable neutrinos
- Annihilation cross-section form
  - $\sigma v = \frac{G^2 s}{16\pi}$ (Dirac type, S-wave)
  - $\sigma v = \frac{G^2 s}{16\pi}\left(1 - \frac{4m^2}{s}\right)$ (Majorana type, P-wave)
- We do not take into account resonance
- $\langle \sigma v \rangle_{app} / \langle \sigma v \rangle_{exact}$:



- Thermally averaged annihilation cross-section
  - $\langle \sigma v \rangle = \frac{G^2 m^2}{16\pi}\left(\frac{12}{x^2} + \frac{5+4x}{1+x}\right)$ (Dirac type, S-wave)
  - $\langle \sigma v \rangle = \frac{G^2 m^2}{16\pi}\left(\frac{12}{x^2} + \frac{3+6x}{(1+x)^2}\right)$ (Majorana type, P-wave)

Solution of the Boltzmann equation

- Freeze out temperature $\Gamma(x_F) = H(x_F)$ where:- $\Gamma(x_F) = \langle \sigma v \rangle n_{eq}(x_F)$ (Different from standard definition of x_F)
- Leads to semi analytical expression for x_f hence computing Ωh^2 possible
- Assume that the comoving relic abundance does not change after decoupling



- Constant g_{*s}
- $g = 2$

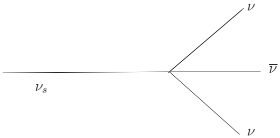
Feasibility of relic densities

- Decoupling at $x_F = 1.8$ and $g_{*s} = 10$ with $\Omega_{DM} h^2 = 0.13$ $\Rightarrow m \sim eV$
  - Too light
- Coupling $G > 1 GeV^{-2}$
  - Not a very promising scenario

Entropy production

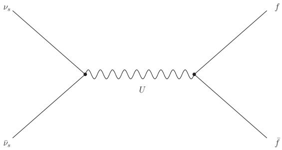
- Out of equilibrium decay produces entropy

$$\frac{s_f}{s_i} = 1.7\, g_*^{1/4} \frac{mY\sqrt{\tau}}{\sqrt{M_{pl}}} \propto \Omega h^2$$



- Sterile neutrinos decay through mixing with standard model neutrinos

$$\Gamma = \frac{1}{\tau} = \frac{G^2 m^2}{192\pi^3} sin^2\theta$$

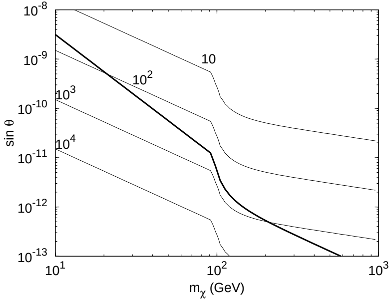


- Large pair annihilation rate via mediation of new U(1) gauge boson

Final results

- Region to the left of the bold line allowed ($\tau < 1$ Sec BBN contraint)
- Possible to produce large entropy

- Entropy production s_f/s_i



Conclusion:-

- Found semi-analytical method to compute density of semi-relativistic relics ($T_F \sim m$)
- Semi-relativistic particles as a stable dark matter relic is not a very promising scenario
- They can be used to produce considerable amount of entropy in the early Universe