A Hypothesis Connecting Dark Energy, Virtual Gravitons, and the Holographic Entropy Bound

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The Holographic Principle

- Idea proposed by t' Hooft and Susskind (mid-1990s), that all of the information contained within a volume is entirely encodable onto a surface bounding that volume.
- Primarily motivated by the oddity that the entropy of a black hole depends on its surface area and not its volume. Bekenstein & Hawking (mid-1970s) showed $S = A_{\mathrm{EH}}/4$ for black holes, which, being of maximally packed energy density, are presumably also of maximally packed entropy density.

Is the holographic principle consistent with vacuum inflation?

- Co-moving volume scales as a^3 while co-moving surface area scales as a^2.
- In this way, the holographic principle appears to be violated by cosmic inflation models that allow portions of the universe to be dominated by $w = -1$ vacuum energy for cosmologically significant timescales.
- For $\rho \propto a^{-3(1+w)}$ to dilute rapidly enough that that the total energy contained within a volume that is co-moving with the expansion of space does not grow faster than the bounding surface area, we must have $3(1 + w) \geq 1$.

Thus, we have **concern if the dark energy equation of state is stiffer than $w = -2/3$** (although temporary false vacuums seem to be allowed if they do not last too long).

But what is an appropriate enclosing surface to pick in cosmology?

- The event horizon? (like a black hole?)

  (but not all cosmologies have an event horizon)
- The particle horizon?

  (not all cosmologies have this either)
- The Hubble volume boundary?
- The apparent horizon?

Event horizon physical properties for black holes and de Sitter cosmologies appear to be *exactly* identical!

Could this be due to identical physics in both cases?

How BH and dS event horizons are the same:

- Once the horizon is crossed, return is impossible.
- Distant observers see time dilation approaching infinity as the horizon is approached. The horizon is not seen to be reached.
- Test particles accelerate toward the horizon.
- Surface acceleration is proportional to total horizon area.
- Distant observers see redshifts approaching infinity as the horizon is approached.
- The horizon bounds a region in which particles can have negative energy with respect to a distant observer. Thus, distant observers should witness a thermal spectrum of particle creation emanating from the horizon.
- (Same entropy also??)

Bousso (2002)

- Demonstrated that the apparent horizon (*not* the event horizon) works as a holographic entropy bound in all physically realistic cases imaginable.

  (Note: The apparent and event horizons coincide in both black holes and in dS cosmologies.)

- Conjectured that the holographic bound might always be saturated at the apparent horizon.

  (Note: In spatially flat cosmologies, the apparent horizon is identical to the Hubble horizon at distance c/H.)

Connection to Curvature?

- In GR, acceleration toward a black hole's EH happens because of spacetime curvature.
- Perhaps objects in our universe accelerate toward our cosmic EH for the same reason?

Old argument from when Einstein's Λ was thought to be zero:

Any contributions to the vacuum energy density predicted by quantum field theories operating in otherwise empty and flat spacetime must be exactly balanced by a "bare" cosmological constant, or some symmetry that relates the quantum contributions to each other is responsible for the cancellation, resulting in an effective $\rho_{vac} = 0$ in the case of zero curvature.

But if flat spacetime should have $\Lambda = 0$*, that would suggest* $|\Lambda| > 0$ *in curved backgrounds.*

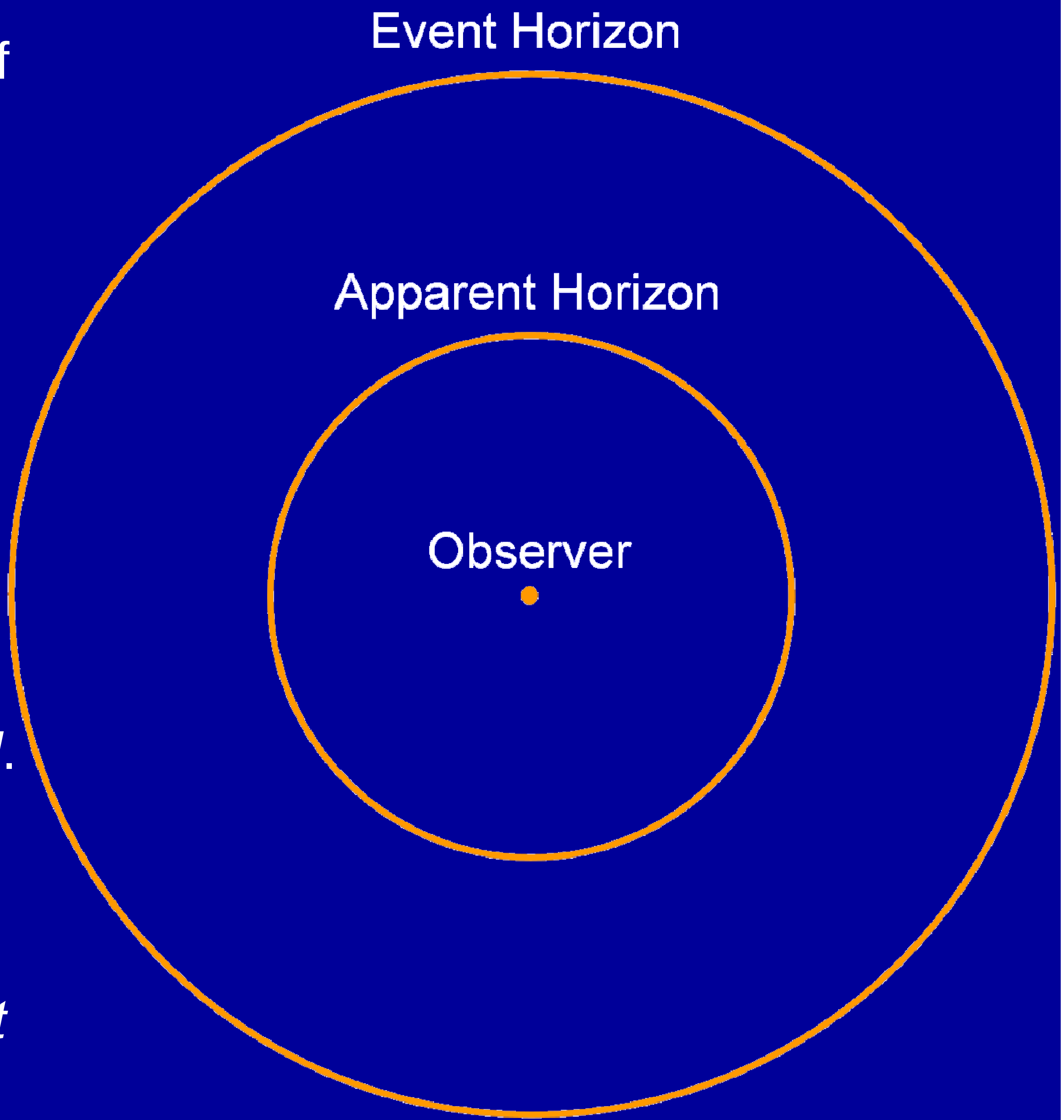
What if the dark energy were related directly to the global curvature, $\Lambda(t) \propto |\Omega_K| \equiv |\Omega_{tot} - 1|$?

- Since $|\Omega_K|$ is just past its maximum in cosmic history (at least since before inflation), $\Lambda(t)$ also would have just passed a maximum value.
- Given the global radius of curvature, $R_C = (c/H)|\Omega_K|^{-1/2}$, the time-dependent $\Lambda(t)$ would maintain proportionality to the *area* of the Hubble apparent horizon (for small Ω_K).
- In general, $|\Omega - 1| \propto (aH)^{-2}$ and, in a flat universe, $H^2 \propto \rho \propto a^{-3(1+w)}$ (ρ and w being of the universe's dominant constituent). $H \propto t^{-1}$ then forces an **effective $w = -2/3$ when $\Lambda(t)$ dominates.**

Relative placement of the apparent and event horizons within a flat universe that is fully dominated by a component with an effective equation of state $P = -(2/3)\rho$, for which $a \propto t^2$ implies an age-Hubble parameter relationship of $t = 2/H$.

$$r_{AH} = c/H = ct/2$$

$$r_{EH} = \int_t^{\infty} (c/a)\,dt' = ct$$



Toy model: Decaying $\Lambda(t) \propto |\Omega_K|$ as the only component in an otherwise empty universe.

- Evolution under eq. of state $w = -2/3$
- Assume approximately flat (small Ω_K)
- Assume homogenous entropy density
- Assume saturated holographic entropy bound within *apparent* horizon, $S_{AH} = A_{AH}/4$
- Given A_{EH} is 4 times greater than A_{AH}, while the volume enclosed by the EH is 8 times greater, deduce that $S_{EH} = A_{EH}/2$
- Assume surface acceleration, κ, of EH is therefore twice as strong as a black hole of equal size, *i.e.*, $\kappa_{EH} = c^2/r_{EH} = c/t$.

Now plug into Einstein's definition:

$$\Lambda \equiv 3\ddot{a}/a$$

For a = r_{EH} = ct, the event horizon surface acceleration sets $\ddot{a} = c^2/r_{EH} = c/t$.

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What is Λ measured to be in our universe today (with $\Omega_\Lambda \cong 0.73$)?

$$(1.2 \pm 0.2) \times 10^{-35}\ \mathrm{s}^{-2} \qquad (!!!)$$

What “stuff” would have the characteristics to display Ω_K dependence and $w = -2/3$?

I suggest extremely low energy “immortal” virtual gravitons.

(Call them IVGs)

If massless virtual particles persist for cosmologically long times, their wavelengths should stretch as $\lambda \propto a$, thus weakening their energies ($E \propto \lambda^{-1}$) such that $\rho \propto a^{-1}$ and $w = -2/3$.

For a massless virtual particle created near the Big Bang to have persisted for t_0 = 13.7 Gyr, its maximum possible energy from the Heisenberg uncertainty condition $E_{max} \sim \hbar/t_0$ would be ~1.5×10^{-33} eV.

The wavelength of such a particle would exceed the Hubble length by a factor of ~6, in apparent violation of boundary conditions.

However, while spin-0 and spin-1 massless particles may be subject to boundary conditions at the cosmological horizon, the Lagrangian of spin-2 particles (gravitons) appears unable to maintain gauge invariance if any boundary conditions are set.

This suggests that virtual gravitons may persist in the universe indefinitely while no other virtual particles are permitted to do so.

These gravitons would eventually dominate over radiation and matter, stimulating an acceleration of space ($w = -2/3 \;\Rightarrow\; a \propto t^2$).

Although virtual, the evolution of their energies, $E \propto a^{-1}$, renders these gravitons immortal in any accelerating universe (in which a increases faster than t).

Critical Density of IVGs and the Holographic Entropy Bound

- If the universe were filled with a critical density, $\rho_{\text{crit}} = 3H_0^2/8\pi G$, of IVGs with average energy $E \sim 5 \times 10^{-34}$ eV, the number of IVGs enclosed within the apparent horizon would be N ~ 10^{122}.
- This is remarkably close to $A_{\text{AH}}/4 \sim 2 \times 10^{122}$ (!)
- Thus, a fixed entropy per graviton of $\mathcal{O}$(a few) would saturate the holographic bound perfectly.
- The number density of IVGs would evolve according to $n_{\text{IVG}} \propto (\rho_{\text{IVG}}/E_{\text{max}}) \propto t^{-1}$.
- Given an AH enclosed volume $\propto t^3$ with surface area $\propto t^2$, the IVGs would always and forever maintain a precise saturation of the holographic limit!