Spherically symmetric solutions of massive gravity

Eugeny Babichev

APC, University Paris 7, Paris

in collaboration with
C. Deffayet and R. Ziour

Motivation

There are pathologies in such theories (ghosts, singular solutions, etc). Why to study them?

- Modification of gravity - a way to get acceleration of the Universe (geometrical Dark Energy).
- To investigate these (relatively) simple models in detail in order to find more complicated theories with no pathologies in them.
- Such massive gravities share some properties with Dvali-Gabadadze-Porrati (DGP) gravity which has also the advantage to produce late time acceleration.

Massive Gravity and Bigravity Theories

- **Pauli-Fierz term,** *Fierz'39; Fierz&Pauli'39*

$$S_m = -\frac{1}{8} m^2 M_P^2 \int d^4x h_{AB} h_{CD} \left(\eta^{AC}\eta^{BD} - \eta^{AB}\eta^{CD}\right)$$

- **Non-linear generalization,**

$$S = \int d^4x \sqrt{-g} \left(\frac{M_P^2}{2} R_g + L_g\right) + S_{int}[f, g]$$

- g **is dynamical**
- f **is flat (non-dynamical)**
- **matter is coupled to** g
- $S_{int}[f, g]$ **is a scalar density under common diffeomorphisms**
- $S_{int}[f, g]$ **takes the PF term when expanded...**

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- **Non-linear generalization,**

$$S = \int d^4x \sqrt{-g} \left(\frac{M_P^2}{2} R_g + L_g \right) + S_{int}[f, g]$$

$$S_{int}^{(2)} = -\frac{1}{8} m^2 M_P^2 \int d^4x \sqrt{-f}\, H_{\mu\nu} H_{\sigma\tau} \left(f^{\mu\sigma} f^{\nu\tau} - f^{\mu\nu} f^{\sigma\tau}\right)$$

Boulware&Deser'72

$$S_{int}^{(3)} = -\frac{1}{8} m^2 M_P^2 \int d^4x \sqrt{-g}\, H_{\mu\nu} H_{\sigma\tau} \left(g^{\mu\sigma} g^{\nu\tau} - g^{\mu\nu} g^{\sigma\tau}\right),$$

Arkani-Hamed, Georgi, Schwartz'03

$$H_{\mu\nu} = g_{\mu\nu} - f_{\mu\nu}$$

Spherically symmetric solutions

$$\boxed{\begin{aligned} g_{\mu\nu}dx^\mu dx^\nu &= -e^{-\nu(R)}dt^2 + e^{\lambda(R)}dR^2 + R^2 d\Omega^2 \\ f_{\mu\nu}dx^\mu dx^\nu &= -dt^2 + \left(1 - \frac{R\mu'(R)}{2}\right)^2 e^{-\mu(R)}dR^2 + e^{-\mu(R)}R^2 d\Omega^2 \end{aligned}}$$

- **GR limit, m=0,**

$$\nu = -\lambda = \ln\left(1 - \frac{R_S}{R}\right)$$

- **Perturbative limit** $m \to 0$ **(and Pauli-Fierz theory)**

$$\nu = -2\lambda$$

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van Damm-Veltman-Zakharov (vDVZ) discontinuity

van Dam, Veltman'70; Zakharov'70

Vainshtein's conjecture

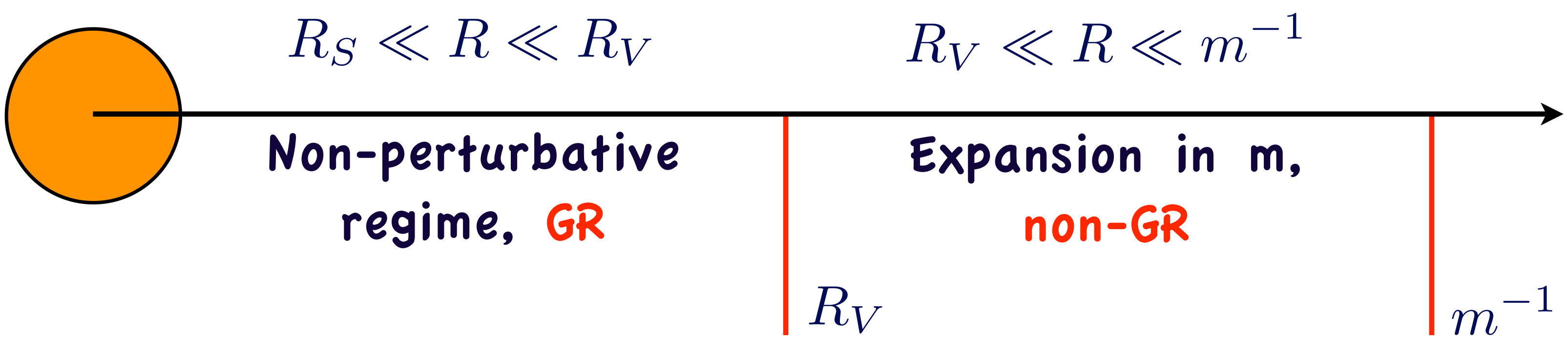
Vainshtein'72

- New scale, the Vainshtein radius,

$$R_V = \left(\frac{R_S}{m^4}\right)^{1/5}$$

In "non-perturbative" regime, one recovers GR values for ν and λ, and

$$\mu \sim \sqrt{\lambda} \sim \sqrt{-\nu} \sim \sqrt{\frac{R_S}{R}}$$



Is it possible to match large R and small R (Vainshtein) solutions???

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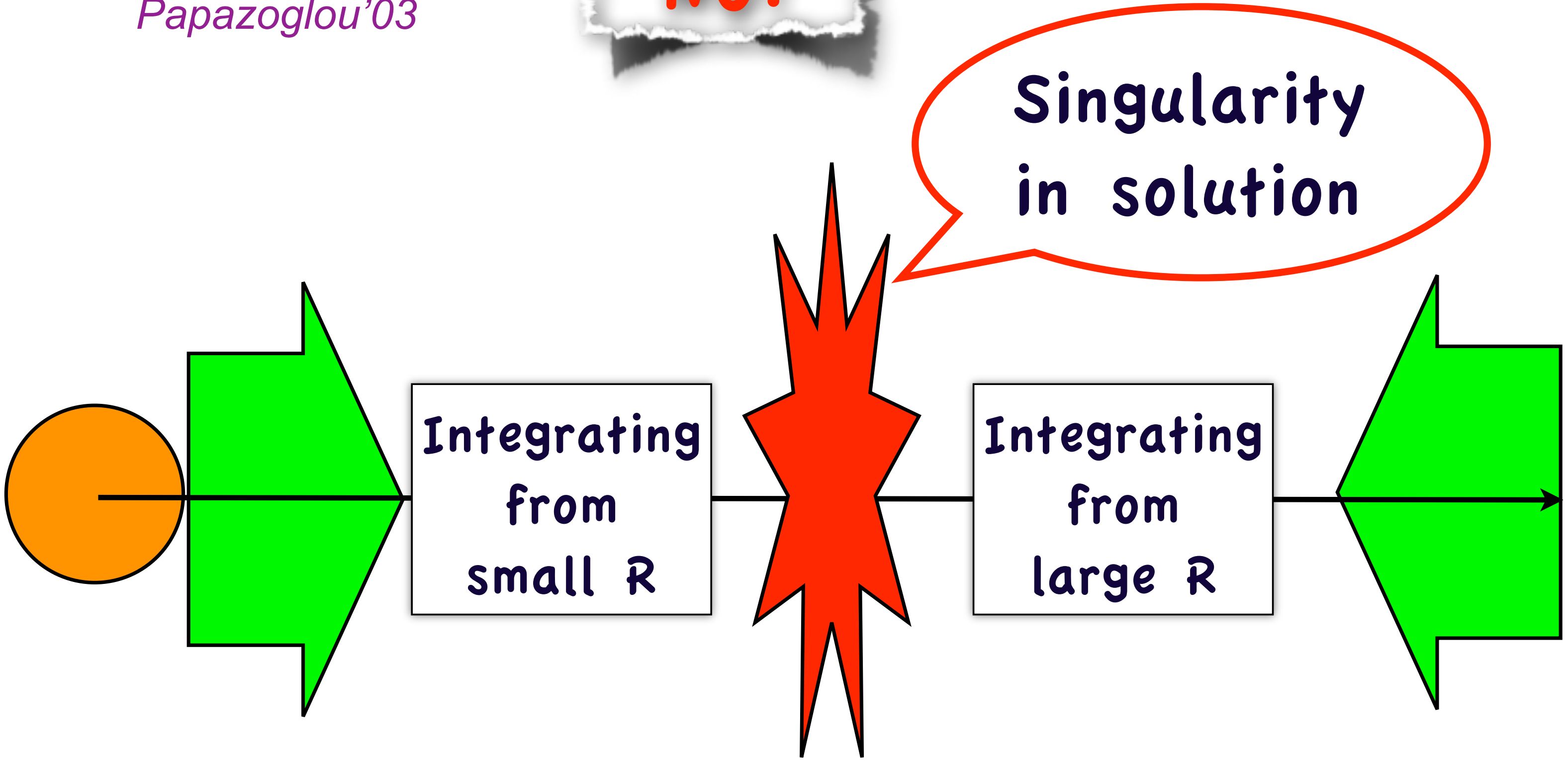
Damour, Kogan, Papazoglou'03

NO!



Decoupling limit

decouple new degrees of freedom from GR ones

$$\Lambda = \left(m^4 M_P\right)^{1/5}$$

$$\boxed{\begin{array}{rcl} M_P & \to & \infty \\ m & \to & 0 \\ \Lambda & \sim & const \\ T_{\mu\nu}/M_P & \sim & const, \end{array}}$$

closely connected to
the Goldstone picture

$$
\begin{aligned}
e^{\nu-\lambda}\left(\frac{\lambda'}{R}+\frac{1}{R^2}(e^{\lambda}-1)\right) &= 8\pi G_N\left(T^g_{tt}+\rho e^{\nu}\right), \\
\frac{\nu'}{R}+\frac{1}{R^2}\left(1-e^{\lambda}\right) &= 8\pi G_N\left(T^g_{RR}+Pe^{\lambda}\right), \\
\nabla^{\mu}T^g_{\mu R} &= 0.
\end{aligned}
$$

$$
\frac{2}{\Lambda^5}Q(\mu,\mu',\mu'')+\frac{3}{2}\,\mu=\frac{C_0}{R^3}
$$

$$
\begin{aligned}
\lambda &= \lambda(\mu) \\
\nu &= \nu(\mu)
\end{aligned}
$$

$$
\begin{aligned}
Q(\mu) &= \frac{1}{2R}\left\{3\alpha\left(6\mu\mu'+2R\mu'^2+\frac{3}{2}R\mu\mu''+\frac{1}{2}R^2\mu'\mu''\right)\right. \\
&\qquad \left.+\beta\left(10\mu\mu'+5R\mu'^2+\frac{5}{2}R\mu\mu''+\frac{3}{2}R^2\mu'\mu''\right)\right\}
\end{aligned}
$$

$$\frac{2}{\Lambda^5}Q(\mu) + \frac{3}{2}\,\mu = \frac{C_0}{R^3}$$

small R

large R

Vainshtein solution,

$$\frac{2}{\Lambda^5}Q(\mu) + \cancel{\frac{3}{2}\,\mu} = \frac{C_0}{R^3}$$

$$\mu \propto \frac{1}{\sqrt{R}}, \qquad (\nu = -\lambda)$$

pertutbative regime,

$$\cancel{\frac{2}{\Lambda^5}Q(\mu)} + \frac{3}{2}\,\mu = \frac{C_0}{R^3}$$

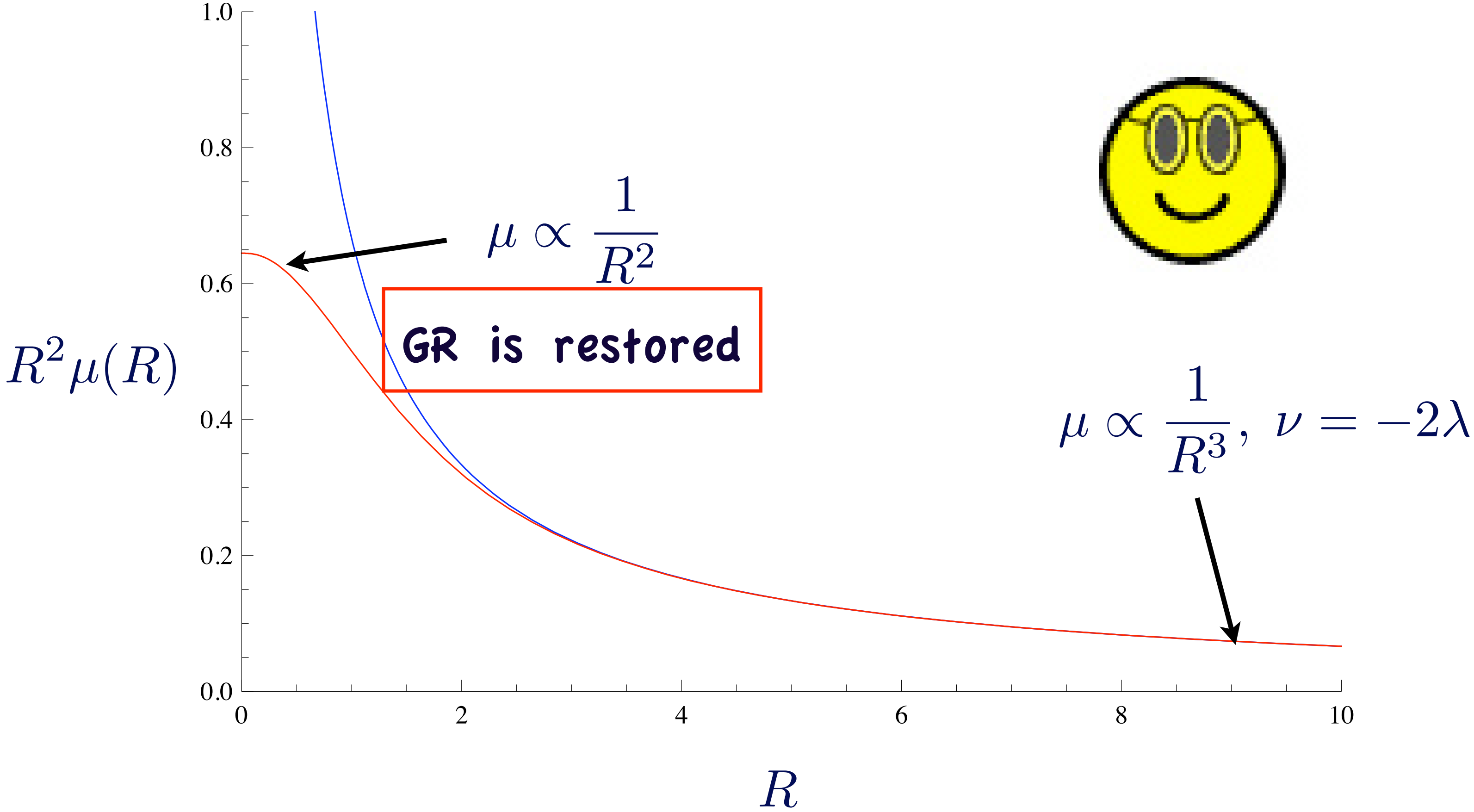
$$\mu \to \frac{2C_0}{3R^3}, \qquad (\nu = -2\lambda)$$

Another solution!

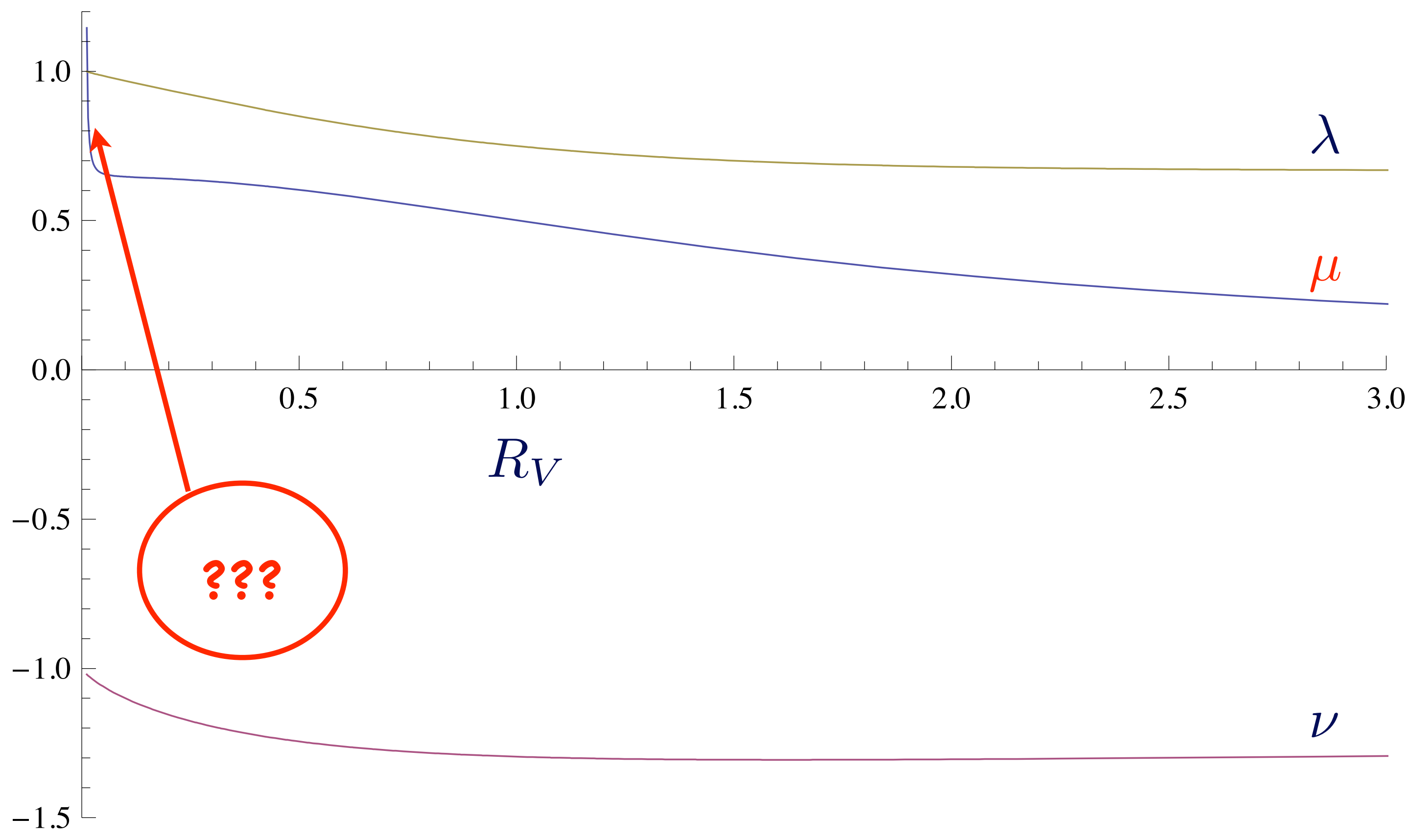
$$\frac{2}{\Lambda^5}Q(\mu) + \cancel{\frac{3}{2}\,\mu} = \cancel{\frac{C_0}{R^3}}$$

$$Q(\mu) = 0 \quad \Rightarrow \quad \mu \propto \frac{1}{R^2}, \ (\nu = -\lambda)$$

Decoupling limit



Full (non-decoupled) system



Full (non-decoupled) system

Need for analytic study!

- The solution for λ, ν, μ of the full system can be found as a series expansion.
- It indicates that this solution is valid only in the range $R \gg R_{new} \equiv R_V^2 m$

- **Vainshtein radius,**

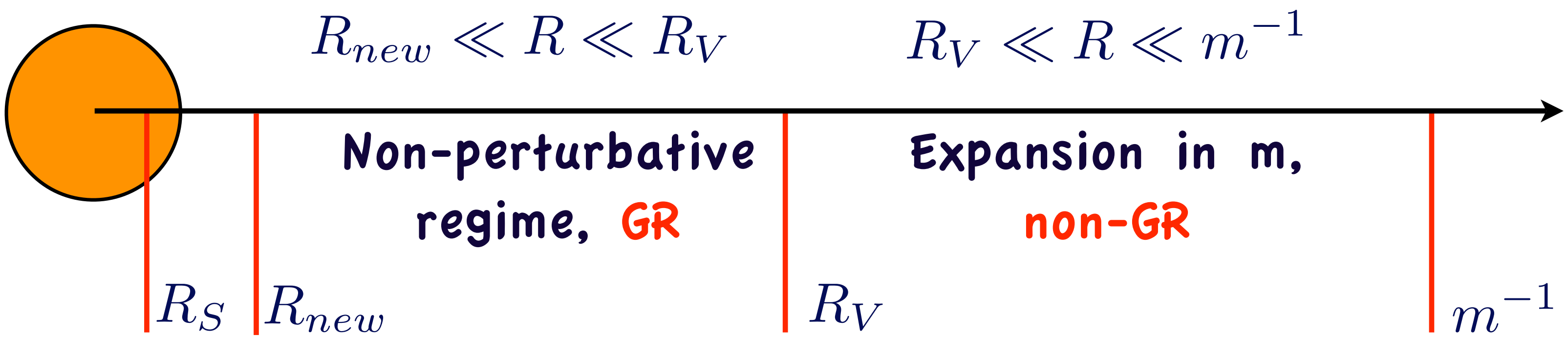
$$R_V = \left(\frac{R_S}{m^4}\right)^{1/5}$$

- **Another scale!**

$$R_{new} = R_V^2 m$$

"non-perturbative" regime,
GR values
for ν and λ

$$\mu \propto \frac{1}{R^2}$$



Conclusion

- The solution proposed by Vainshtein does not continue to an asymptotically flat solution in the decoupling limit.
- There is another solution which can be smoothly extended to an asymptotically flat solution and is associated with zero modes of the non-linearities appearing in the decoupling limit.
- For the full non-linear system our new scaling seems to break down at some R_{new}.
- This leaves open the possibility that there is a non-singular solution, though, with mass terms different from those we have investigated.
- The decoupling limit is missing important features of non-linear massive gravity.