Cosmological Helium production and WIMP dark matter in modifications of gravity

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Introduction

Combined observational data from CMB, SN, LSS indicate that
i) Universe expands in accelerating rate
ii) Most of the energy density of the universe is in a dark form (dark matter, dark energy)
Within GR one must introduce extra fields to play the role of dark matter (e.g. superpartners) and dark energy (e.g. dynamical scalar field)
Alternative: Explain dark side of the universe by modifying gravity

Consider the model

$$S = \int d^4x\sqrt{-g}f(R) + S_{matter} \quad (1)$$

Varying w.r.t. metric we obtain the field eqns for gravity (generalized Einstein's eqns)

$$G_{\mu\nu} = T_{\mu\nu}^{matter} + T_{\mu\nu}^{grav} \quad (2)$$

The second term in total energy-momentum tensor comes from gravity itself and can play the role of dark energy.

In addition: One can compute the gravitational potential in the non-relativistic limit

$$V(r) = \frac{1}{r} + \delta V(r) \quad (3)$$

The modification in the gravitational potential can explain the galaxy rotation curves without dark matter.

The model

We wish to investigate primordial nucleosynthesis and WIMP dark matter within the simple model

$$f(R) \sim R^{n} \tag{4}$$

where n is the parameter of the model and the special value $n = 1$ corresponds to GR. First obtain the field eqns

$$f' R_{\mu\nu} - \frac{1}{2} f g_{\mu\nu} - \nabla_{\mu} \nabla_{\nu} f' + g_{\mu\nu} \Box f' = \kappa^{2} T_{\mu\nu}, \tag{5}$$

where $T_{\mu\nu}$ is the energy-momentum tensor for the matter.
For gravity we consider the spatially flat Robertson-Walker line element

$$ds^{2} = dt^{2} - a(t)^{2}(dx^{2} + dy^{2} + dz^{2}), \tag{6}$$

For matter we consider a cosmological fluid ($\rho(t)$, pressure $p(t)$)

$$T^{\mu}_{\nu} = diag(\rho, -p, -p, -p) \tag{7}$$

Using the cosmological equations it is possible to obtain exact simple solution for the early universe (radiation era)

$$a(t) \sim t^{n/2} \tag{8}$$

and

$$H(T) \sim T^{2/n} \tag{9}$$

Nucleosynthesis

Consider temperatures $T \leq 100$ MeV so that nucleons exist. Define $\Delta m = m_n - m_p = 1.29$, $y = \Delta m/T$, $\tau \simeq 886$ sec, and

$$X_n = \frac{n_n(T)}{n_n(T) + n_p(T)} \tag{10}$$

The cosmological helium abundance is given by

$$Y_4 = 2exp(-t_c/\tau)X(T \simeq 0) \tag{11}$$

where $t_c \sim 3$ min corresponds to the temperature (1/25 of deuterium binding energy or 100 keV) at which deuterium can form helium

$$exp(B/T_c)\eta \sim 1 \tag{12}$$

The frozen value $X(T \simeq 0)$ is computed solving the basic rate equation (I.C. $X(y=0)=1/2$)

$$\frac{dX(t)}{dt} = \lambda_{pn}(t)(1 - X(t)) - \lambda_{np}(t)X(t) \tag{13}$$

where λ_{pn}: processes protons into neutrons
and λ_{np}: processes neutrons into protons.
From particle physics

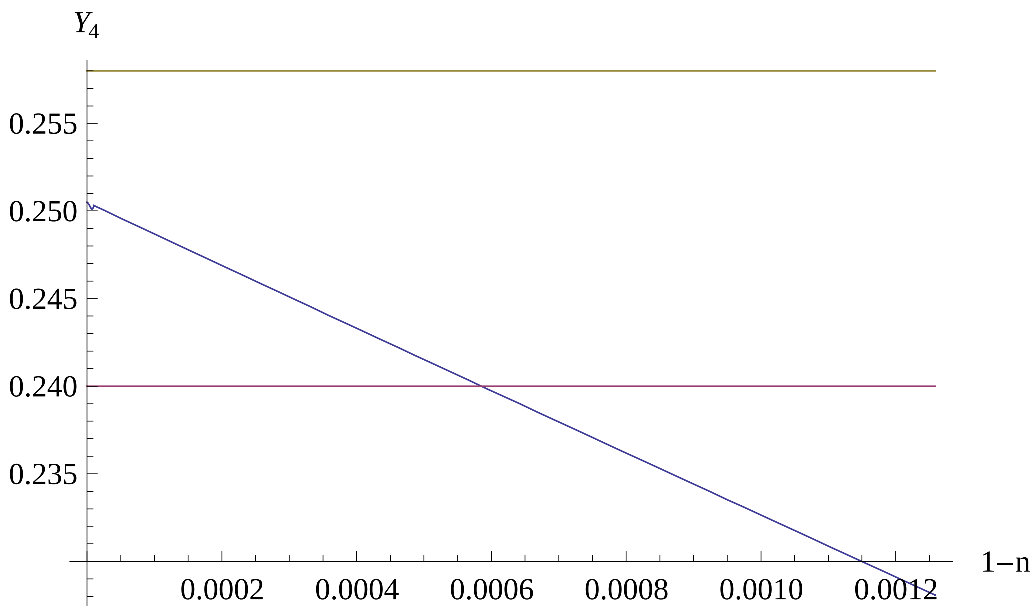
$$\lambda_{np}(y) = \left(\frac{255}{\tau y^5}\right)(12 + 6y + y^2) \tag{14}$$

and

$$\lambda_{pn}(y) = e^{-y}\lambda_{np}(y) \tag{15}$$

In terms of y

$$\frac{dX(y)}{dy} = \frac{dt}{dy}(\lambda_{pn}(y)(1 - X(y)) - \lambda_{np}(y)X(y)) \tag{16}$$



Theoretical helium 4 abundance versus $\delta = 1 - n$. The strip shows the allowed observational range.

Dark (WIMP) matter

Assume that a weakly interacting particle χ of mass $m \sim 100$ GeV plays the role of DM in the universe

Must compute the abundance and impose the DM constraint

$$\Omega_\chi h^2 = \Omega_{dm} h^2 \sim 0.1 \tag{17}$$

Standard method: Integrate Boltzmann eqn

$$\dot{n} + 3Hn = - < \sigma v > \ (n^2 - n_{EQ}^2) \tag{18}$$

Introduce new dimensionless quantities

$$x = \frac{m}{T} \tag{19}$$

$$Y = \frac{n}{s} \tag{20}$$

Using

$$H(T) = 1.67 g_*^{1/2} T^2 / M_p \quad (21)$$

$$< \sigma v > = \sigma_0 x^{-l} \quad (22)$$

the Boltzmann equation takes the final compact form

$$\frac{dY}{dx} = -\lambda x^{-l-2} (Y^2 - Y_{EQ}^2) \quad (23)$$

Finally thermal relic abundance for WIMP is given by

$$\Omega_\chi h^2 = \Omega_{cdm} h^2 = \frac{m Y_\infty s(T_0) h^2}{\rho_{cr}} \quad (24)$$

$$Y_\infty \equiv Y(x = \infty) = \frac{l+1}{\lambda} x_f^{l+1} \quad (25)$$

where $x_f \simeq 22$ determined by $H \sim \Gamma$

In the s-wave approximation ($l = 0$) and for a typical cross section $\sigma_0 \sim \alpha^2/M_{ew}^2$ one obtains $\Omega_\chi h^2 \sim 0.1$.
Now take into account the modifications of gravity

$$\tilde{l} = l + (2 - \frac{1}{\alpha}) \tag{26}$$

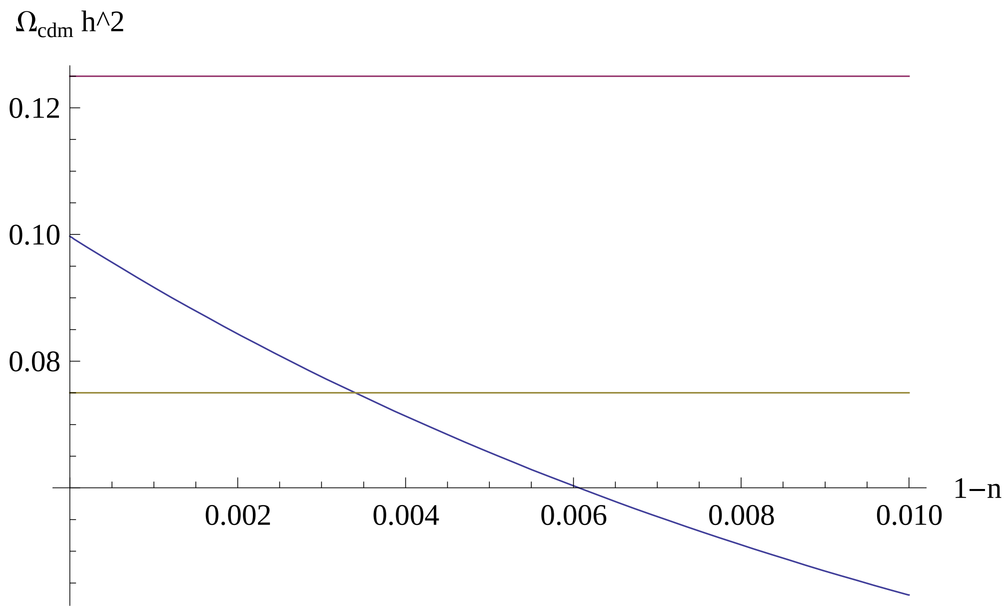
$$\tilde{\sigma}_0 = \frac{H(m)}{H_\alpha(m)}\sigma_0 \tag{27}$$

where

$$H_\alpha(m) = \frac{\alpha A^{\frac{1}{2}}}{g_\alpha^{\frac{1}{4\alpha}} M_p^{\frac{1}{2\alpha}}} \left(\frac{4\pi^3 g_*}{15}\right)^{\frac{1}{4\alpha}} m^{\frac{1}{\alpha}} \tag{28}$$

and

$$H(m) \equiv H_{\alpha=1/2}(m) \tag{29}$$



Neutralino relic density versus $\delta = 1 - n$. The strip shows the allowed observational range.

Conclusions

- We have discussed a class of modifications of gravity, $f(R) \sim R^n$
- This class of models predict a new expansion law for the early universe, $a(t) \sim t^{n/2}$
- BBN and WIMP dark matter considerations constrain the single model parameter n
- Our investigation shows that the bound coming from BBN is more stringent, and $n \simeq 1$
- The main result is that the class of models under discussion is only slightly different than Einstein's general relativity

Thank you