On stars in f(R) gravity models

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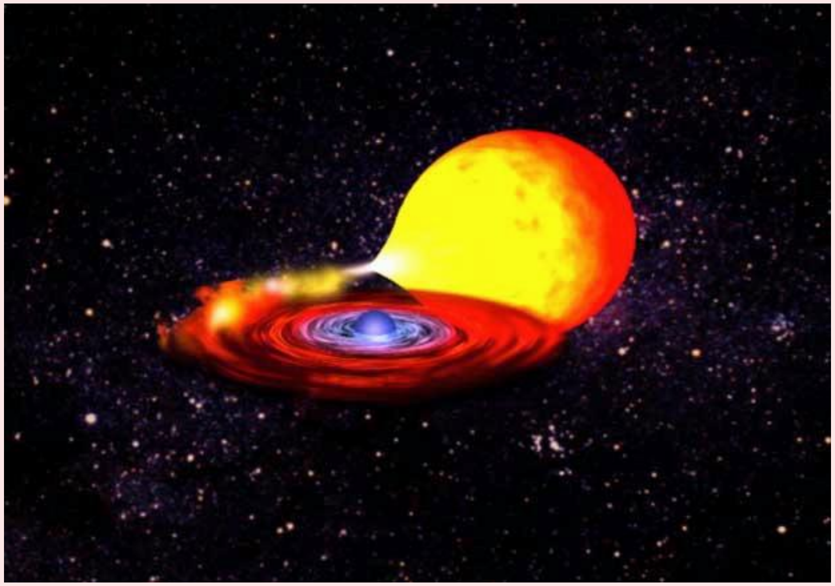
Stars in f(R) gravity models

If you modify
General Relativity,

what happens to stars?

Drastic changes in structure?

...Observations?



f(R) models

- $f(R)$ theories:

$$S = \frac{1}{16\pi G}\int d^4x\sqrt{-g}f(R) + \int d^4x\sqrt{-g}\mathcal{L}_m, \tag{1}$$

- **GR:** $R - 2\Lambda \rightarrow$ **A general function of R:** $f(R)$

- E.g. $f(R) = R - \mu^4/R$

- Could construct models using other Lorentz invariant quantities ($R_{\mu\nu}R^{\mu\nu}, \nabla_\mu R \nabla^\mu R$, ...)

- $f(R)$ models are a subclass of scalar-tensor theories

f(R) models

- Why study extensions to GR?
  - Observations (dark energy, dark matter)
  - Theory (GR an effective theory of gravity)
  - Just because it's interesting (How "stable" is GR?)

- Inflation models (Starobinsky 1980), eg. $f(r) = R + \alpha R^2$

- "First papers on cosmological models in $f(R)$ gravity appeared already in 1969-1970" (Starobinsky: *Disappearing cosmological constant in f(R) gravity*, arXiv: 0706.2041)

f(R) models

- $f(R)$ as "geometrical dark energy" – modifying the gravity sector, not the matter sector

- DE: the expansion of the universe seems to be accelerating
  ...a huge amount of suggestions giving good $a(t)$
  $\rightarrow$ reason some out by supplementary investigations!

  (E.g. contradicting structure formation, or what is done here)

- $f(R) = R - \frac{\mu^4}{R} \rightarrow$ "the simplest correction which becomes important at extremely low curvatures"

  (Carroll, Duvvuri, Trodden, Turner: *Is cosmic speed-up due to new gravitational physics?*, arXiv: astro-ph/0306438)

- Palatini vs. metric formalism
  - **Palatini:** the connection $\Gamma^{\rho}_{\mu\nu}$ defining the Riemann tensor

$$R^{\rho}_{\sigma\mu\nu} = \partial_{\mu}\Gamma^{\rho}_{\nu\sigma} - \partial_{\nu}\Gamma^{\rho}_{\mu\sigma} + \Gamma^{\rho}_{\mu\lambda}\Gamma^{\Lambda}_{\nu\sigma} - \Gamma^{\rho}_{\nu\lambda}\Gamma^{\lambda}_{\mu\sigma} \tag{2}$$

    and the metric $g_{\mu\nu}$ are both free dynamical variables
  - **Metric formalism:** $g_{\mu\nu}$ a free variable, the Levi-Civita connection

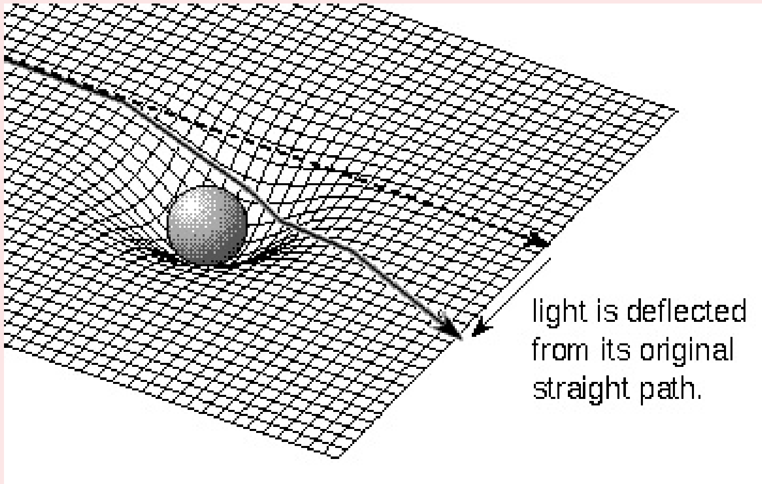
$$\Gamma^{\rho}_{\mu\nu} = \frac{1}{2}g^{\rho\sigma}(\partial_{\mu}g_{\sigma\nu} + \partial_{\nu}g_{\sigma\mu} - \partial_{\sigma}g_{\mu\nu}) \tag{3}$$

  - Test particles fall along the extremal (shortest, "straight") paths on the manifold = the affine geodesics wrt the Levi-Civita connection

Spherically symmetric solutions

- Cosmological expansion history $\longrightarrow$ the modified Friedmann equations

  – But (how) does the Schwarzschild solution change? The interior solution, the star?



Spherically symmetric solutions

- The general static, spherically symmetric metric + star: perfect fluid

$$ds^2 = -e^{A(r)}dt^2 + e^{B(r)}dr^2 + r^2 d\Omega^2$$

$$+ \; T^\mu_\nu = \mathrm{diag}(-\rho(r), p(r), p(r), p(r))$$

- To determine

the metric $(A(r), B(r))$

+ the stellar structure $(\rho(r), p(r))$

need to solve the gravitational field equations

= **the (MODIFIED) Einstein equation**

Schwarzschild- de Sitter and TOV

- GR:
  - $R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G T_{\mu\nu} - \Lambda g_{\mu\nu}$
  - $\nabla_\mu T^{\mu\nu} = 0$
  - **Exterior** ($T_{\mu\nu} = 0, r \geq R$): The Schwarzschild- de Sitter solution

$$ds^2 = -\left(1 - \frac{2GM}{r} - \frac{1}{3}\Lambda r^2\right) dt^2 + \frac{dr^2}{1 - \frac{2GM}{r} - \frac{1}{3}\Lambda r^2} + r^2 d\Omega^2$$

  - **Interior** ($r \leq R$): The Tolman-Oppenheimer-Volkoff (TOV) = eq. of the hydrostatic equilibrium inside the star

$\frac{dp(r)}{dr} = -\frac{(\rho(r)+p(r))(Gm(r)+4\pi G r^3 p(r))}{r(r-2Gm(r))}$, where $m(r) = \int_0^r dr 4\pi r^2 \rho(r)$

Schwarzschild- de Sitter and TOV

- The mass parameter m(r) defined

$$(g_{rr} =)e^{B(r)} = \frac{1}{1-2Gm(r)/r}$$

- The Schwarzschild mass M = m(R)

- The equation $m(r) = \int_0^r dr 4\pi r^2 \rho(r)$ comes from the Einstein equation

- M the mass of an object ($M \neq \int_0^R dr e^{B(r)/2} 4\pi r^2 \rho(r)$!)

Spherical solutions in the f(R) theories: Metric f(R) gravity

- The modified Einstein equation – vary the action wrt the metric

$$FR_{\mu\nu} - \frac{1}{2} f g_{\mu\nu} - \nabla_\mu \nabla_\nu F + g_{\mu\nu} \Delta^\alpha \Delta_\alpha F = 8\pi G T_{\mu\nu},$$

notation: $F(R) \equiv \frac{\partial f(R)}{\partial R}$, eg. $f(R) = R - \mu^4/R \to F(R) = 1 + \mu^4/R^2$

- $\nabla_\mu T^{\mu\nu} = 0$
- The trace: $\nabla_\mu \nabla^\mu F + \frac{1}{3}(FR - 2f) = \frac{8\pi G}{3} T$ – **cf. GR:** $R = -8\pi GT + 4\Lambda$

Spherical solutions in the f(R) theories: Metric f(R) gravity

- In the spherically symmetric, static case ($' \equiv \frac{d}{dr}$)

$$A' = -\frac{1}{1 + rF'/2F}\left(\frac{1 - e^B}{r} - \frac{re^B}{F}8\pi Gp + \frac{re^B}{2}\left(R - \frac{f}{F}\right) + \frac{2F'}{F}\right)$$

$$B' = \frac{1 - e^B}{r} + \frac{re^B}{F}\frac{8\pi G}{3}(2\rho + 3p) + \frac{re^B}{6}\left(R + \frac{f}{F}\right) - \frac{rF'}{2F}A'.$$

Spherical solutions in the f(R) theories: Palatini f(R) gravity

- The modified Einstein equation – vary the action wrt $g_{\mu\nu}$ and $\Gamma^{\rho}_{\mu\nu}$:

$$FR_{\mu\nu} - \frac{1}{2} f g_{\mu\nu} = 8\pi G T_{\mu\nu}$$

$$\nabla_{\rho}(\sqrt{-g} F g^{\mu\nu}) = 0$$

- $\tilde{\nabla}_{\mu} T^{\mu\nu} = 0$, where $\tilde{\nabla}_{\mu}$ is the covariant derivative wrt. the Levi-Civita
- The trace: $FR - 2f = 8\pi G T$ (GR: $R = -8\pi G T + 4\Lambda$)

- In the spherically symmetric, static case

$$A' = -\frac{1}{1 + rF'/2F}\left(\frac{1 - e^B}{r} - \frac{e^B}{F} 8\pi G r p + \frac{\alpha}{r}\right)$$

$$B' = \frac{1}{1 + rF'/2F}\left(\frac{1 - e^B}{r} + \frac{e^B}{F} 8\pi G r \rho + \frac{\alpha + \beta}{r}\right)$$

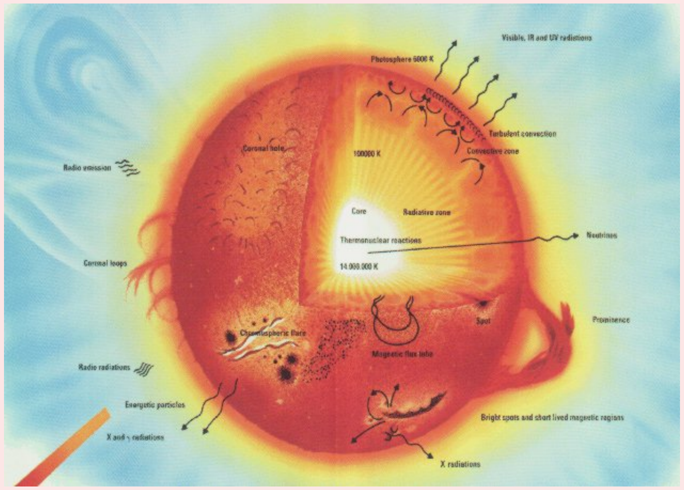
where

$$\alpha = r^2 \left(\frac{3}{4}\left(\frac{F'}{F}\right)^2 + \frac{2F'}{rF} + \frac{e^B}{2}\left(R - \frac{f}{F}\right)\right)$$

$$\beta = r^2 \left(\frac{F''}{F} - \frac{3}{2}\left(\frac{F'}{F}\right)^2\right) \qquad \leftarrow F'' \propto T''!!!$$

- The continuity equation $p' = -\frac{A'}{2}(\rho + p)$

Solar System observations



- The structure and microphysics of the Sun well known (interior solution)
- So far the experiments (exterior solution) give **upper bounds** for deviations from GR

- **No contradiction to GR** predictions has been observed (except the Pioneer anomaly)

Solar System observations

- E.g. Solar System observations constrain the value of the cosmological constant (Kagramanova, Kunz, Lammerzahl: *Solar system effects in Schwarzshild-de Sitter spacetime*, arXiv: gr-qc/0602002):

Observed effect	Estimate on Λ
gravitational redshift	$\lvert\Lambda\rvert \leq 10^{-27} m^{-2}$
perihelion shift	$\lvert\Lambda\rvert \leq 10^{-41} m^{-2}$
light deflection	no effect
gravitational time delay	$\lvert\Lambda\rvert \leq 6 \cdot 10^{-24} m^{-2}$
geodetic precession	$\lvert\Lambda\rvert \leq 10^{-27} m^{-2}$
Pioneer anomaly	$\Lambda \sim -10^{-37} m^{-2}$

- To account for dark energy $\Lambda \sim 10^{-52} m^{-2}$

The Post-Newtonian parameters

- The amount of deviations from Newtonian theory in Solar System scale gravity effects ("weak field limit")

- The Post-Newtonian parameters β_{PPN} and γ_{PPN}:

$$ds^2 = -\left(1 - \frac{2GM}{r} + \frac{\beta_{PPN}}{2}\left(\frac{2GM}{r}\right)^2\right) dt^2 + \left(1 + \gamma_{PPN}\frac{2GM}{r}\right)(dr^2 + r^2 d\Omega^2$$

- $\beta_{PPN} = 1, \gamma_{PPN} = 1$ in GR

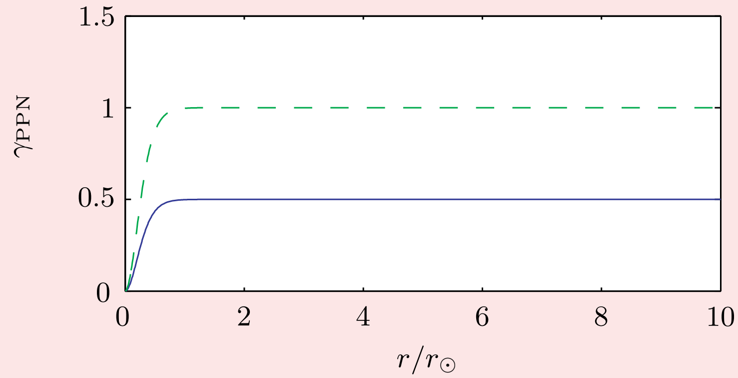
- **Experiments:** Lunar Laser Ranging $\beta_{PPN} - 1 \leq 2.3 \cdot 10^{-4}$

  Cassini Tracking $\gamma_{PPN} - 1 \leq 2.3 \cdot 10^{-5}$

The weak field limit in f(R) models

- Kainulainen, Reijonen, Sunhede: *The interior spacetimes of stars in Palatini f(R) gravity*. **arXiv: gr-qc/0611132**; Kainulainen, Piilonen, Reijonen, Sunhede: *Spherically symmetric spacetimes in f(R) gravity theories*. **arXiv: 0704.2729**

Numerical results: the Sun + $f(R) = R - \mu^4/R$



- Palatini f(R): $\gamma_{PPN} = 1$
- Metric f(R): $\gamma_{PPN} = 1/2$(!!!)*

*) except for a tuned class of solutions $\longrightarrow$ the Dolgov-Kawasaki instability, see Kainulainen, Sunhede: *On the stability of spherically symmetric spacetimes in metric f(R) gravity*, arXiv: 0803.0867

Compact objects in modified gravity theories

- Compact objects: white dwarfs, neutron stars and black holes

  – the final stages of stellar evolution

- Small size, enormous densities

  – strong gravitational fields, advanced microphysics

- **Equilibrium structure** and **stability**:

  f(R) (or other alternative gravity models) vs. GR

- Binary star dynamics; rotation; magnetic fields; gravitational waves; supernovae; ...

Compact objects in modified gravity theories

- The "death" of a star: no more nuclear fuel to burn $-$ no more support by thermal pressure against gravitational collapse
  - White dwarfs: the pressure of degenerate electrons
  - Neutron stars: $\sim$ the pressure of degenerate neutrons

- Degenerate Fermi gas; $T = 0$ single species of ideal (non-interacting) fermions

$$p = \frac{2}{3h^3}\int_0^{p_F} dp 4\pi p^2 \frac{p^2c^2}{\sqrt{p^2c^2+m_x^2c^4}}$$

- The **polytropic** equation of state

$$p = K\rho_0^\gamma$$

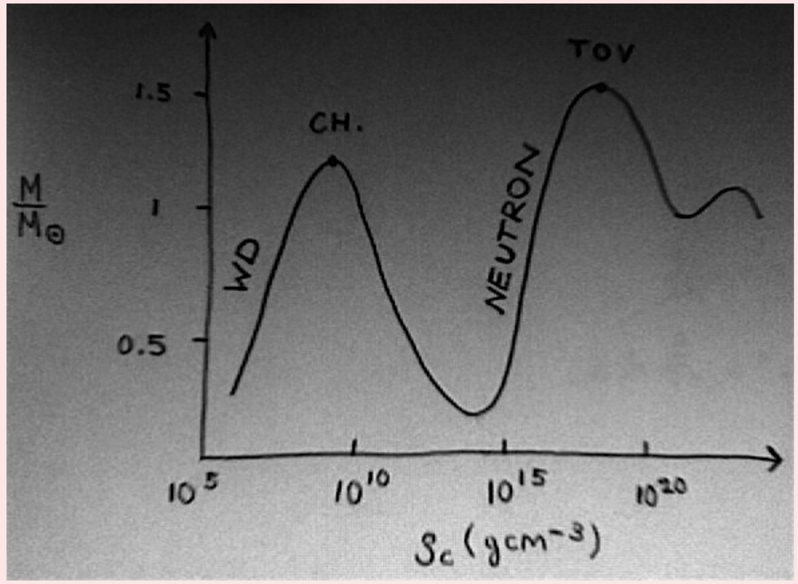
K, γ constants and $\rho_0 = n_x m_x$ the rest mass density

- Extremely relativistic fermions: $\gamma = 4/3$

Non-relativistic fermions: $\gamma = 5/3$

(electrons $\rho_0 << 10^9$ kg/m^3 (ions), neutrons $\rho_0 << 6 \cdot 10^{18}$ kg/m^3)

- Corrections: electrostatic interactions – onset of inverse β-decay $e^- + p \rightarrow n + \nu_e$ – nucleon interactions – relativistic strongly interacting matter – quark matter...a bit "messy" = complicated + not-so-well-known physics



CH. = **The Chandrasekhar limit**, maximum mass of a white dwarf

TOV = **The TOV limit**, maximum mass of a neutron star

- E.g. If the Chandrasekhar limit became a bit smaller, might the supernovae Ia appear a bit dimmer ($\Delta E_B \sim \frac{GM_{core}^2}{R}$)?

- E.g. Palatini $f(R) = R - \frac{\mu^4}{R}$
  $\rightarrow F(R) = 1 + \frac{\mu^4}{R^2}$

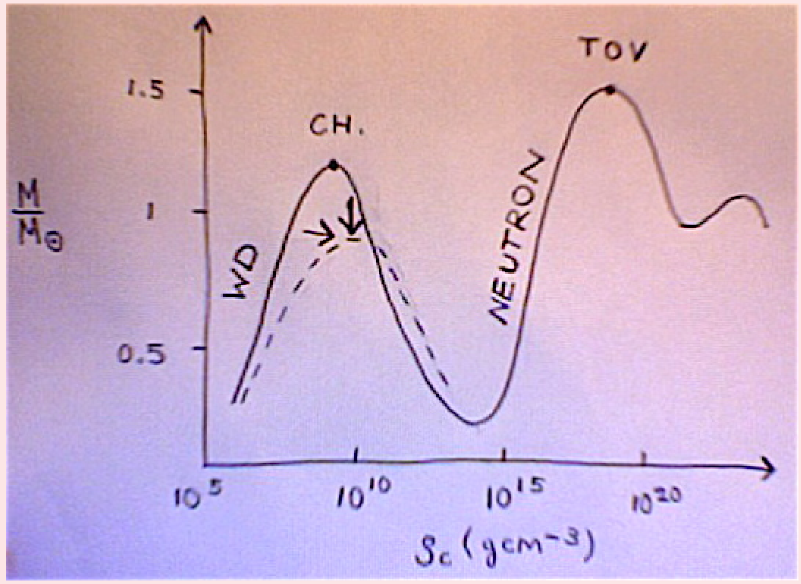
- From the trace equation $R = \frac{1}{2}(-8\pi GT + \sqrt{(8\pi GT)^2 + 12\mu^4})$ $(T = 3p - \rho)$

- Take $\rho(r) \sim$ constant:

  $$B' = \frac{1}{1+rF'/2F}\left(\frac{1-e^B}{r} + \frac{e^B}{F}8\pi Gr\rho + \frac{\alpha+\beta}{r}\right) \approx \frac{1-e^B}{r} + \frac{e^B}{F}8\pi Gr\rho$$

  $$\rightarrow M = \int_0^R dr 4\pi r^2 \frac{\rho(r)}{F}$$

- **i)** $|T| >> \mu^2 \rightarrow F \approx 1 \rightarrow M = M_{GR}$
  ii) $|T| << \mu^4 \rightarrow R = \sqrt{3\mu^4} \rightarrow F = \frac{4}{3} \rightarrow M = \frac{3}{4}M_{GR}$



- The Chandrasekhar limit:
  $E = E_F + E_G$
  "A given $\rho(r) \longrightarrow$ less E_G than in GR" $\longrightarrow$ the peak shifts to the right, to higher ρ_c – a **more dense** wd explodes as SnIa
  "E_G grows faster than E_F as a function of ρ" $\longrightarrow$ the peak is lower: the maximum mass of a wd becomes lower – **less energy** is released in SnIa

- Dark energy $\mu_{DE}^2 \sim 10^{-26}$ kg/m^3

- A Newtonian star (eg. the Sun): $\rho >> p \to T \approx -\rho >> \mu_{DE}^2 \to$ case i) $M = M_{GR}$

- To worry ($M \neq M_{GR}$...), should have $p = \frac{1}{3}\rho$ (extremely relativistic)

Conclusion

- See a forthcoming paper...