Field-theoretical branes and their effective actions

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work in collaboration
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(in preparation)

UniverseNet, The second network school and meeting, 22-26 September 2008

Outline

- Motivation - why this calculation?

  Branons and the fate of the zero mode of the 5D kink in the presence of gravity

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- Effective action of a domain wall

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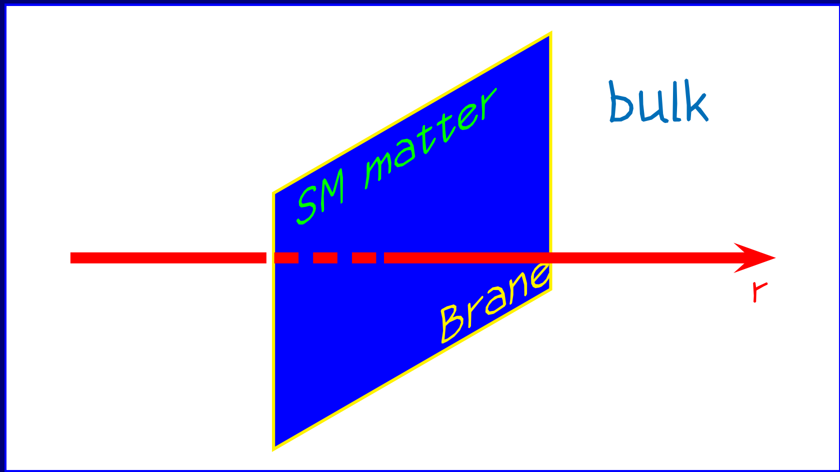
  Branons and the fate of the zero mode of the 5D kink in the presence of gravity
- Effective action of a domain wall
- Conclusions

FT brane-world models

Rubakov and Shaposhnikov, 1983
Akama, 1983

- Idea: Our universe is a ``brane'': a (3+1)-dimensional defect in a higher-dimensional field theory:



Thin branes – approximations of finite-width defects

SM particles: low-energy modes trapped on the defect ⇒ extra dimensions only visible in the very high energy experiments; 4D action – low energy effective action

Branons

Goldstone bosons of broken isometries of extra space

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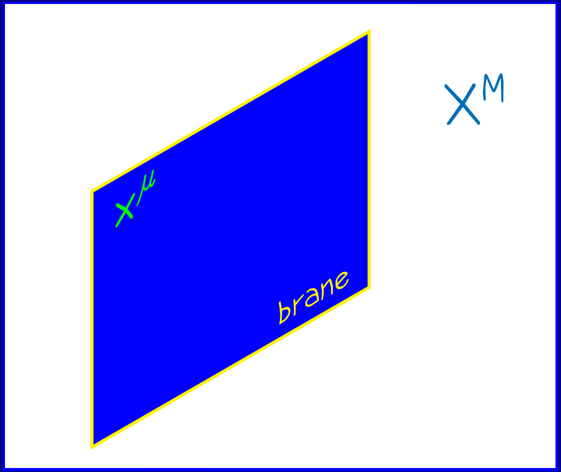

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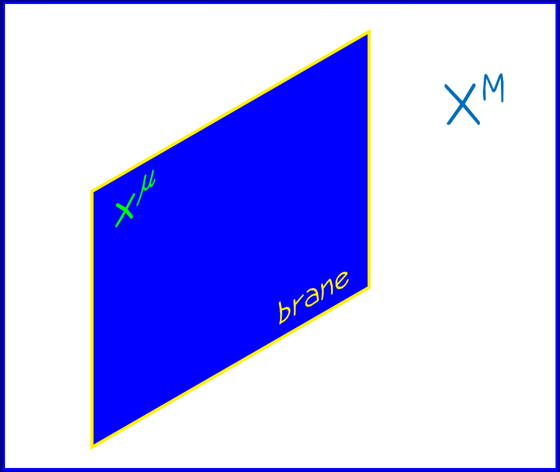
- brane (static gauge): $\begin{cases} X^\mu(x) = x^\mu \\ X^5(x) = Y(x) \end{cases}$
- induced metric: $g_{\mu\nu} = \eta_{\mu\nu} - \partial_\mu Y \partial_\nu Y$



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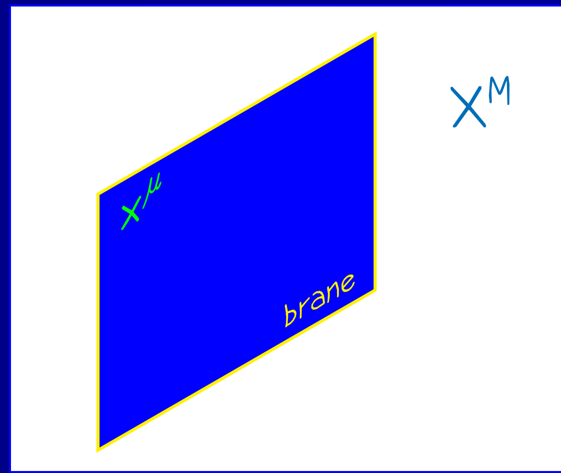

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- induced metric: $g_{\mu\nu} = \eta_{\mu\nu} - \partial_\mu Y \partial_\nu Y$
- Nambu-Goto action ($\mathcal{T}$ tension, branon $\tilde{Y}(x) = \sqrt{\mathcal{T}} Y(x)$)

$$S_{\text{brane}} = -\int d^4x \sqrt{-g}\, \mathcal{T}$$

$$= \int d^4x \left\{ -\mathcal{T} + \frac{1}{2}\partial_\mu \tilde{Y} \partial^\mu \tilde{Y} + \frac{1}{8\mathcal{T}} (\partial_\mu \tilde{Y} \partial^\mu \tilde{Y})^2 + \ldots \right\}$$

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Branon ~ zero mode of the 5D kink

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Our study: gravitational backreaction of the brane important

wide resonance $\Longleftrightarrow$ high probability to tunnel into the bulk

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$\Rightarrow$ Look into the effective action of the brane perturbations!

Domain wall – two scalars model

Action: $$S = \int d^4x dy \left[\frac{1}{2}\eta^{MN}\partial_M\Phi\partial_N\Phi + \frac{1}{2}\eta^{MN}\partial_M\Xi\partial_N\Xi - V(\Phi,\Xi)\right],$$

$$V(\Phi,\Xi) = \frac{\lambda}{4}\left(\Phi^2 - v^2\right)^2 + \frac{\tilde{\lambda}}{4}\Xi^4 + \frac{1}{2}M^2\Xi^2 + \frac{1}{2}\alpha(\Phi^2 - v^2)\Xi^2$$

Idea: Set up a brane as a domain wall. Compute the 4d low energy effective action.

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If $M^2 < \alpha v^2$ and $\lambda\tilde{\lambda}v^4 > (\alpha v^2 - M^2)^2$, the system has a degenerate GS ($\Phi_{GS} = \pm v, \Xi_{GS} = 0$)

$\Rightarrow$ domain wall interpolating between the two vacua;

kink configuration ($\Phi = v \tanh(ay), \Xi = 0$), $a^2 = \lambda v^2/2$, always solves EOM

Perturbations around ($\Phi = v\tanh(ar), \Xi = 0$)

Look at the perturbations around the background (using 4D Poincaré invariance):

$$\Phi(x,y) = \Phi_c(y) + \varphi(x^\mu, y) = \Phi_c(y) + \sum\!\!\!\!\!\!\int_n \; f_n(y)u_n(x)$$

$$\Xi(x,y) = \xi(x^\mu, y) = \sum\!\!\!\!\!\!\int_n \; h_n(y)v_n(x)$$

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- $u_n(x)$, $v_n(x)$ – scalar fields from the 4D point of view
- $f_n(y)$, $h_n(y)$ – wave functions; determine the localization of the modes on the brane:

$$\begin{cases} -\partial_y^2 f + \left(4a^2 - \dfrac{6a^2}{\cosh^2(ar)}\right) f = m^2 f \\ -\partial_y^2 h + \left(M^2 - \dfrac{\alpha v^2}{\cosh^2(ar)}\right) h = \tilde{m}^2 h \ . \end{cases}$$

Spectrum of perturbations

Lowest lying states:

kink's zero mode: $\psi_0 = f_0(y)u_0(x) = N_0 \frac{va}{\cosh^2(ay)} u_0(x)$

massive mode of Ξ: $\psi_1 = h_1(y)v_1(x) = N_1 \frac{va}{\cosh^{\sigma}(ay)} v_1(x)$

$$\sigma = \frac{1}{2}\left(-1 + \sqrt{1 + 8\frac{\alpha}{\lambda}}\right)$$

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- mass of $v_1(x)$ is $\tilde{m}_1^2 = -\sigma^2 a^2 + M^2$

$\Rightarrow$ if we choose $M^2 = +\sigma^2 a^2 + \frac{\lambda v^2}{4}\epsilon^2$, $|\epsilon| \ll 1$

we get a light v_1: $\tilde{m}_1^2 = (\lambda v^2/4)\epsilon^2$

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- rest of the spectrum:

for u_n's: heavy mode with mass $3a^2$, continuum from $4a^2$

for v_n's: continuum starts at $M^2 \approx \sigma a^2$, if other localized modes, then their masses $\mathcal{O}(a^2)$

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$a \gg \tilde{m}_1 \Rightarrow$ we can construct a sensible 4D low energy action

Towards the effective action...

Presence of a zero mode ⇒ introduce a collective coordinate :

$$\Phi = \Phi_c(y - Y(x)) + \sum\!\!\!\!\!\!\int_{n\neq 0} f_n(y - Y(x))\, u_n(x)$$

$$\Xi = \xi(x,y) = h_1(y)v_1(x) + \sum\!\!\!\!\!\!\int_{n\neq 1} h_n(y - Y(x))\, v_n(x)$$

⇒ only derivative interactions for Y, as expected for a Goldstone.

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Effective action? Easy - neglect all the terms involving the heavy modes (only corrs to couplings, suppressed by the heavy scale):

$$S_{eff} = \int d^4x \left\{ -\mathcal{T} + \frac{1}{2}\partial^\mu \tilde{Y} \partial_\mu \tilde{Y} + \frac{1}{2}\partial^\mu v_1 \partial_\mu v_1 - \frac{1}{2}\tilde{m}^2 v_1^2 \right.$$
$$\left. - \frac{\tilde{\lambda}}{4}\left(\int_{-\infty}^{\infty} dy\, h_1^4\right) v_1^4 + \frac{1}{2\mathcal{T}}\left(\int_{-\infty}^{\infty} dy\, h_1'^2\right) v_1^2 \partial^\mu \tilde{Y} \partial_\mu \tilde{Y} \right\}$$

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Really?

Integrating out the heavy modes

Wrong! The heavy modes contribute significantly because of the trilinear interactions LLH even when $m_n \to \infty$ and cannot be simply thrown away!

S. Ranjbar-Daemi, A. Salvio, M. Shaposhnikov, `On the decoupling of heavy modes in Kaluza-Klein theories' Nucl.Phys.B741:236-268,2006.

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We have to integrate them out:

$$S_H = \int d^4x \left\{ \frac{1}{2}\partial^\mu H \partial_\mu H - \frac{1}{2} m_H^2 H^2 + JH \right\}$$

Effective action:

$$S_{eff} = -\frac{1}{2} \int d^4x d^4y \; J(x) \Delta_H(x-y) J(y) = \frac{1}{2M_H^2} \int d^4x \; J^2(x) + \ldots$$

Importance of the heavy modes

Integrating out the heavy modes:

$$
\begin{aligned}
S_{\mathrm{eff}} = \int d^4x \Bigg\{ & -\mathcal{T} + \frac{1}{2}\partial^\mu \tilde{Y} \partial_\mu \tilde{Y} + \frac{1}{2\mathcal{T}^2} \sum\!\!\!\!\!\!\!\int_{n\neq 0} \frac{1}{m_n^2} \left(\int_{-\infty}^{\infty} dy\, \Phi_c' f_n' \right)^2 \left(\partial^\mu \tilde{Y} \partial_\mu \tilde{Y} \right)^2 \\
& + \frac{1}{2} \partial^\mu v_1 \partial_\mu v_1 - \frac{1}{2} \tilde{m}^2 v_1^2 \\
& - \left[\frac{\tilde{\lambda}}{4} \left(\int_{-\infty}^{\infty} dy\, h_1^4 \right) - \frac{1}{2\mathcal{T}^2} \sum\!\!\!\!\!\!\!\int_{n\neq 0} \frac{1}{m_n^2} \left(\int_{-\infty}^{\infty} dy\, \Phi_c' f_n' \right)^2 \right] v_1^4 \\
& + \frac{2}{\mathcal{T}} \sum\!\!\!\!\!\!\!\int_{n\neq 1} \frac{1}{\tilde{m}^2} \left(\int_{-\infty}^{\infty} dy\, h_1 h_n' \right)^2 \partial^\mu \tilde{Y} \partial^\nu \tilde{Y} \partial_\mu v_1 \partial_\nu v_1 \\
& + \left[\frac{1}{2\mathcal{T}} \int_{-\infty}^{\infty} dy\, h_1'^2 - \frac{\alpha}{\mathcal{T}} \sum\!\!\!\!\!\!\!\int_{n\neq 0} \frac{1}{m_n^2} \left(\int_{-\infty}^{\infty} dy\, \Phi_c' f_n' \right) \left(\int_{-\infty}^{\infty} dy\, \Phi_c' f_n' \right) \right] v_1^2 \partial^\mu \tilde{Y} \partial_\mu \tilde{Y} \Bigg\}
\end{aligned}
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$$+ \frac{1}{2}\partial^\mu v_1 \partial_\mu v_1 - \frac{1}{2}\tilde{m}^2 v_1^2$$

$$- \left[\frac{\tilde{\lambda}}{4}\left(\int_{-\infty}^{\infty} dy\, h_1^4\right) - \frac{1}{2\mathcal{T}^2}\sum\!\!\!\!\!\!\int_{n\neq 0} \frac{1}{m_n^2}\left(\int_{-\infty}^{\infty} dy\, \Phi_c' f_n'\right)^2\right] v_1^4$$

$$+ \frac{2}{\mathcal{T}}\sum\!\!\!\!\!\!\int_{n\neq 1} \frac{1}{\tilde{m}^2}\left(\int_{-\infty}^{\infty} dy\, h_1 h_n'\right)^2 \partial^\mu\tilde{Y}\partial^\nu\tilde{Y}\partial_\mu v_1 \partial_\nu v_1$$

$$+ \underbrace{\left[\frac{1}{2\mathcal{T}}\int_{-\infty}^{\infty} dy\, h_1'^2 - \frac{\alpha}{\mathcal{T}}\sum\!\!\!\!\!\!\int_{n\neq 0} \frac{1}{m_n^2}\left(\int_{-\infty}^{\infty} dy\, \Phi_c' f_n'\right)\left(\int_{-\infty}^{\infty} dy\, \Phi_c' f_n'\right)\right]}_{=\,0!} v_1^2 \partial^\mu\tilde{Y}\partial_\mu\tilde{Y} \Big\}$$

Connection with NG action

We expect our effective action:

$$S_{eff} = \int d^4x \left\{ -\mathcal{T} + \frac{1}{2}\partial^\mu\tilde{Y}\partial_\mu\tilde{Y} + \frac{1}{8\mathcal{T}}\left(\partial^\mu\tilde{Y}\partial_\mu\tilde{Y}\right)^2 + \frac{1}{2}\partial^\mu v_1\partial_\mu v_1 - \frac{1}{2}\tilde{m}^2 v_1^2 - \left(\frac{\tilde{\lambda}}{4} - \frac{\lambda\sigma^2}{16}\right)\left(\int_{-\infty}^{\infty} dy\, h_1^4\right) v_1^4 + \frac{1}{2\mathcal{T}}\partial^\mu\tilde{Y}\partial^\nu\tilde{Y}\partial_\mu v_1\partial_\nu v_1 \right\}$$

to coincide with the Nambu-Goto action:

$$S_{NG} = \int d^4x\sqrt{-g}\left\{-\mathcal{T} + \frac{1}{2}\partial^\mu v_1\partial_\mu v_1 - \frac{1}{2}\tilde{m}^2 v_1^2 - \frac{\lambda_4}{4}v_1^4\right\}$$

$$= \int d^4x\left\{-\mathcal{T} + \frac{1}{2}\partial^\mu\tilde{Y}\partial_\mu\tilde{Y} + \frac{1}{8\mathcal{T}}\left(\partial^\mu\tilde{Y}\partial_\mu\tilde{Y}\right)^2 + \frac{1}{2}\partial^\mu v_1\partial_\mu v_1 - \frac{1}{2}\tilde{m}^2 v_1^2 - \frac{\lambda_4}{4}v_1^4 + \frac{1}{2\mathcal{T}}\partial^\mu\tilde{Y}\partial^\nu\tilde{Y}\partial_\mu v_1\partial_\nu v_1 - \frac{1}{4\mathcal{T}}\partial^\mu\tilde{Y}\partial_\mu\tilde{Y}\partial^\nu v_1\partial_\nu v_1 + \frac{\tilde{m}_1^2}{4\mathcal{T}}\partial^\mu\tilde{Y}\partial_\mu\tilde{Y}v_1^2 + \ldots\right\}$$

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$$- \left(\frac{\tilde{\lambda}}{4} - \frac{\lambda \sigma^2}{16}\right)\left(\int_{-\infty}^{\infty} dy\, h_1^4\right) v_1^4 + \frac{1}{2\mathcal{T}}\partial^\mu \tilde{Y} \partial^\nu \tilde{Y} \partial_\mu v_1 \partial_\nu v_1 \Big\}$$

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The two actions are almost identical – but not exactly...

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$$S_{eff} - S_{NG} = \int d^4x \left\{ -\frac{\pi^2 - 6}{48 a^2 \mathcal{T}} \partial^\mu \tilde{Y} \partial_\mu \tilde{Y} \partial^\alpha \partial^\nu \tilde{Y} \partial_\alpha \partial_\nu \tilde{Y} \right.$$

$$\left. + \frac{1}{4\mathcal{T}} \left(2F(\sigma) + 1 \right) \partial^\mu \tilde{Y} \partial_\mu \tilde{Y} \left[\partial^\nu v_1 \partial_\nu v_1 - \tilde{m}_1^2 v_1^2 \right] \right\}$$

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Nambu-Goto action is only correct in the zero-thickness approximation!
⇒ curvature corrections

B. Carter, R. Gregory, "Curvature corrections to dynamics of domain walls", PRD51:5839-5846,1995

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No closer to NG... Why?

Nambu-Goto action is only correct in the zero-thickness approximation!
⇒ curvature corrections

B. Carter, R. Gregory, "Curvature corrections to dynamics of domain walls", PRD51:5839-5846,1995

Are branon interactions modified by curvature effects?

Geometric description

Yes! As for NG, the effects of branons can be rewritten in purely geometric terms:

$$S_{eff} = \int d^4x\sqrt{-g}\left\{-\mathcal{T} - \frac{(\pi^2-6)\mathcal{T}}{24a^2}R + \frac{1}{2}\partial^\mu v_1\partial_\mu v_1 - \frac{1}{2}\tilde{m}^2 v_1^2 - \frac{\lambda_4}{4}v_1^4 - \frac{1}{4}\left(1+2F(\sigma)\right)v_1^2 R\right\},$$

where $R = -\frac{1}{\mathcal{T}}\partial^\alpha\partial^\nu\tilde{Y}\partial_\alpha\partial_\nu\tilde{Y}$ is the Ricci scalar

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- Comment 1: $\int d^4x\, R \sim \int d^4x \Big\{ \underbrace{\partial^\alpha\left(\partial^\nu\tilde{Y}\partial_\alpha\partial_\nu\tilde{Y}\right)}_{\text{total derivative}} - \underbrace{\partial^\nu\tilde{Y}\partial_\nu\partial^\alpha\partial_\alpha\tilde{Y}}_{\text{vanishes on shell}} \Big\}$,

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- Comment 2: Curvature, but no graviton! Only one d.o.f

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- There are corrections to the Nambu-Goto action which can be expressed through the Ricci scalar of the induced metric
- These corrections are likely to be important to describe branon interactions correctly!
- Including bulk metric perturbations? To be looked into soon. Expect scalar-tensor gravity.