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Loop Quantum Cosmology holonomy corrections to inflationary models

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Fenomenology of loop corrections

• Backgroud evolution in the FRW cosmology with the holonomy
loop correction

• The evolution of the scalar metric perturbations

• The effective comoving sonic horizon

• The power spectrum of the initial energy density perturbations

Ashtekar variables in the FRW Universe: $c = \gamma \dot{a}$ $p = a^2$ $\{c, p\} = \dfrac{\gamma}{3}$

The hamiltonian $H = -\dfrac{3\,N}{\gamma^2} \sqrt{|p|}\, c^2 + H_{mat}$ gives us Friedmann equations
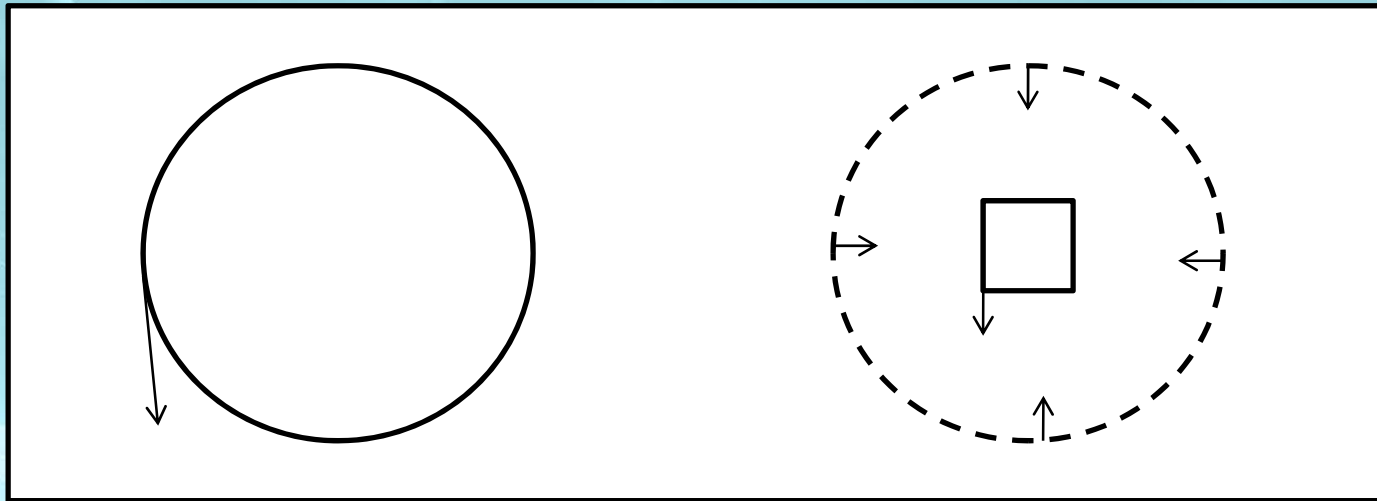
The parallel transport around the loop changes the vector. If we would shrink the loop to the smallest possible size we would get the elementary correction

By considering the loop quantum gravity modifications to the c we get

$$c \rightarrow \sin(cl_j/\sqrt{p})\frac{\sqrt{p}}{l_j}$$, where l_j is the quantum of length.

The holonomy loop correction does not changes p!

$$l_j \propto l_{pl}(j(j+1))^{1/4}$$ ⟹ This is an extremely important variable!

No specific value of j chosen by nature! $j = \frac{1}{2}, 1, \frac{3}{2}, ...$

$$\rho_{cr} = \frac{3}{\gamma^2 l_j^2} \propto \frac{M_{pl}^4}{\sqrt{j(j+1)}}$$ ⟸ Critical (maximal) energy density of the Universe

For $8\pi G = 1$ and big values of j we have $\rho_{cr} \sim 1/j$

Friedmann equations

$$H^2 = \frac{\rho}{3}\left(1 - \frac{\rho}{\rho_{cr}}\right)$$

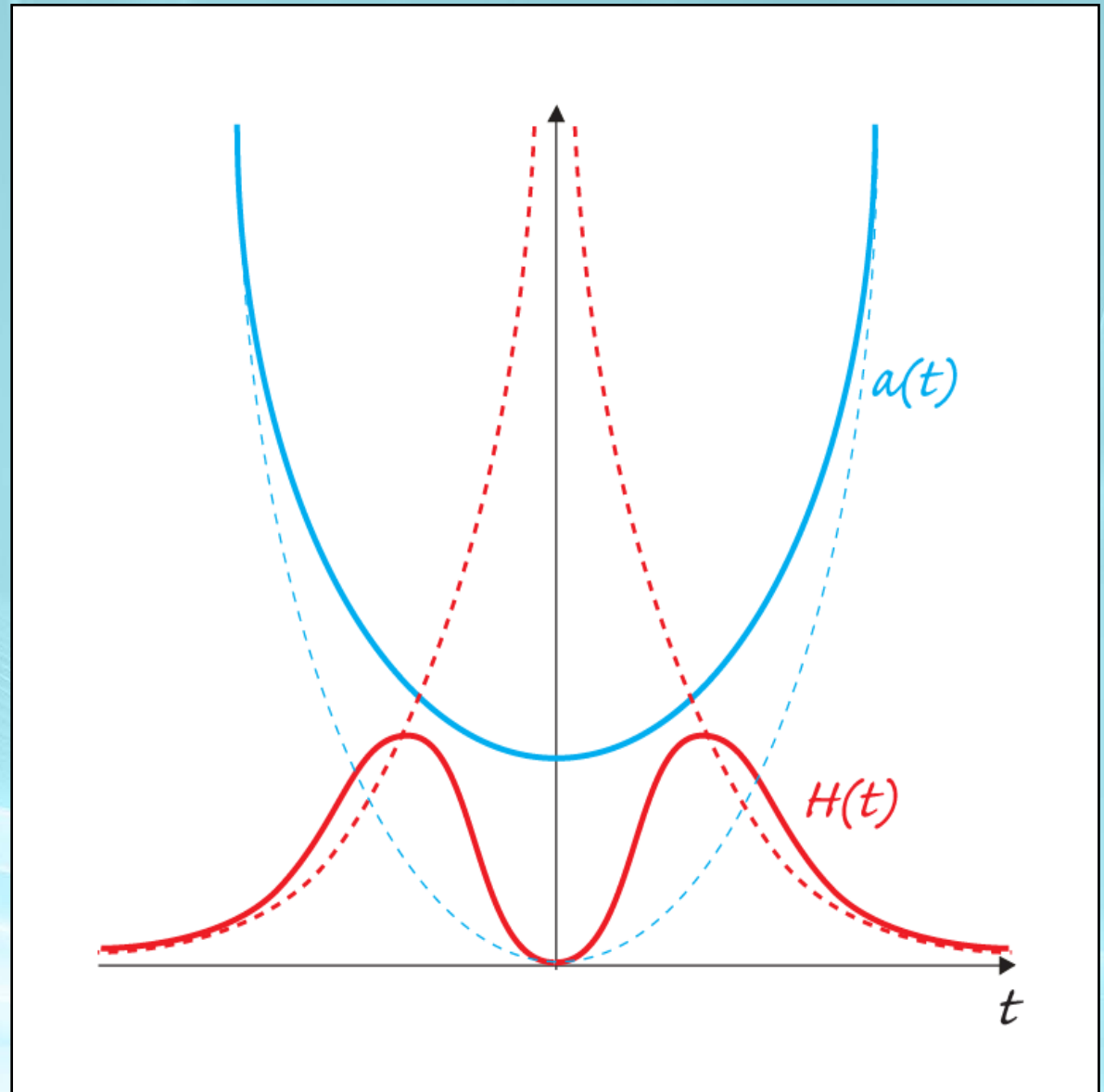$$\dot{H} = -\frac{1}{2}(\rho + P)\left(1 - 2\frac{\rho}{\rho_{cr}}\right)$$

$$\rho/\rho_{cr} \rightarrow 0 \Rightarrow \text{ normal FRW}$$

Effective variables

$$\frac{\rho_{eff}}{3} = \frac{\rho}{3}\left(1 - \frac{\rho}{\rho_{cr}}\right)$$

$$P_{eff} = P\left(1 - 2\frac{\rho}{\rho_{cr}}\right) - \frac{\rho^2}{\rho_{cr}}$$

We can write Hamilton equations for $ds^2 = -N^2 dt^2 + p(d\vec{x})^2$, where

$$p = a^2(1 + 2\Psi)$$

$$N = (1 - \Phi)$$

We consider $k \to 0$ so perturbations are functions of time only. We do not have any anisotropic pressure $\Rightarrow \Phi = \Psi$ and from perturbated friedmann equations we have

$$3H\dot{\Psi} + 3H^2\Psi = -\frac{1}{2}\delta\rho_{eff}$$

$$\ddot{\Psi} + 4H\dot{\Psi} + 3H^2\Psi = \frac{1}{2}\delta P_{eff}$$

For the adiabatic perturbations we obtain

$$\ddot{\Psi} + (4 + 3c_{s_{eff}}^2)H\dot{\Psi} + [2\dot{H} + 3H^2(1 + c_{s_{eff}}^2)]\Psi = 0$$

This equation is almoust identical with the one from the standard FRW. Perturbations are frozen outside the effective sonic horizon.

In LQC the effective speed of sound becomes infinite for $\rho = \rho_{cr}/2$

$$c^2_{s_{eff}} = \frac{\delta P_{eff}}{\delta \rho_{eff}} = c^2_s - 2\frac{(\rho + P)/\rho_{cr}}{1 - 2\rho/\rho_{cr}}$$

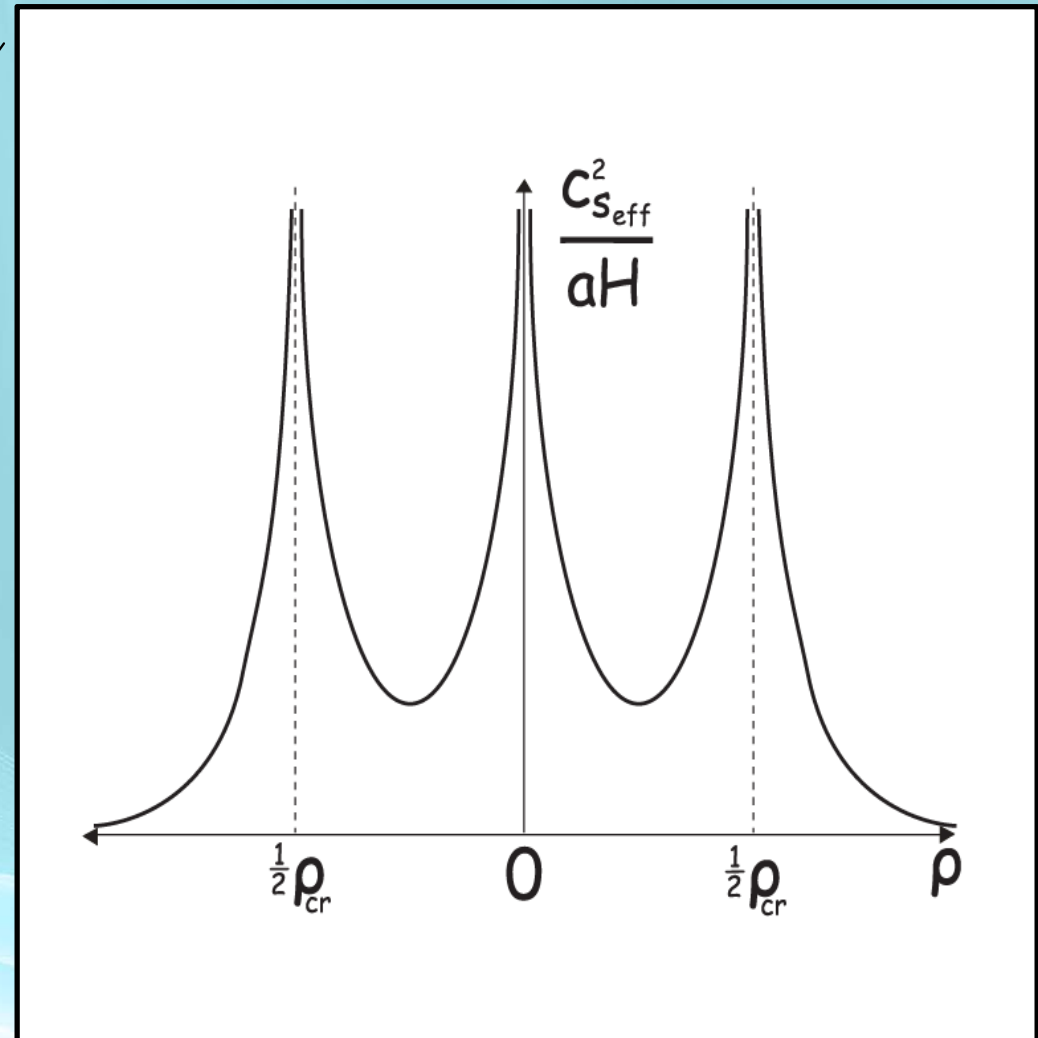No conserved information left over from the $\dot{H} > 0$ period

The effective Big Bang scenario!

$c^2_{s_{eff}}$ is not a physical velocity!!!

Equations for the inflaton and it's perturbation are not changed by the loop correction

$$\ddot{\delta\varphi} + 3H\dot{\delta\varphi} + [V'' + k^2/a^2]\delta\varphi \cong 0$$ where $\delta\varphi$ is the inflaton perturbation

For the slow-roll approximation $$\varepsilon = \frac{1}{2(1-\rho/\rho_{cr})}\frac{V'^2}{V^2}, \qquad V_{eff} = V(1 - V/\rho_{cr})$$

The power spectrum of the curvature perturbations is then in form of

$$\mathcal{P}_{\mathcal{R}_{loop}} = \frac{V_{eff}}{72\pi^2\epsilon} = (1 - \frac{V}{\rho_{cr}})^2 \mathcal{P}_{\mathcal{R}}$$

From the COBE normalisation we know, that if we want to avoid fine tuning we need to have $$\rho_{cr} > (10^{16}\,\text{GeV})^4$$

This gives us the limit for j. For big values of j $$\rho_{cr} \propto M_{pl}^4/j \Rightarrow j < 10^{12}$$

The spectral index $n_s(k) - 1 = 2\eta - 6\varepsilon$ is changed by the LQC to $$n_{s_{loop}}(k) - 1 = \frac{n_s(k) - 1}{(1 - V/\rho_{cr})}$$

- The FRW Universe in the low energy limit for LQC

- No information about the scalar perturbations can cross the ρ_{cr} energy regime untouched

- We can limit the quantum of length by the COBE normalisation

- More models fit's the data for strong LQC holonomy efects