Trans-Planckian relics in the scalar to tensor ratio

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A question

Inflation provides an explanation—and a causal one—for the origins of what we see in the universe

But is its theoretical framework sufficiently sound?

How does inflation make stuff?

Two ingredients:

1) A quantum field, $\varphi(\eta,\mathbf{x})$

2) an expanding universe

$$ds^2 = a^2(\eta)\,[d\eta^2 - d\mathbf{x}\cdot d\mathbf{x}]$$

The origin of structures (inhomogeneities) lies in the fact that quantum fields always fluctuate,

$$\langle 0(\eta)\,|\,\varphi(\eta,\mathbf{x})\,\varphi(\eta,\mathbf{y})\,|\,0(\eta)\rangle \neq 0$$

and the expansion makes tiny stuff big.

A quick sketch in de Sitter space

Let us simplify:

1) A massless free field, $\nabla^2 \varphi = 0$

2) de Sitter space

$$a(\eta) = -1/H\eta\ ,\quad \text{with } \eta = -\infty \ldots 0$$

An operator expansion,

$$\varphi(\eta,\mathbf{x}) = \int \frac{d^3\mathbf{k}}{(2\pi)^3} \left[\ \varphi_k(\eta)\ e^{i\mathbf{k}\cdot\mathbf{x}}\ a_{\mathbf{k}} + \varphi_k{}^{\star}(\eta)\ e^{-i\mathbf{k}\cdot\mathbf{x}}\ a_{\mathbf{k}}{}^{\dagger}\ \right]$$

with the *general* solution,

$$\varphi_k = \frac{H}{\sqrt{2k^3}} \frac{1}{\sqrt{1 - f_k f_k^{\star}}} \left\{ (i - k\eta) e^{ik\eta} - f_k (i + k\eta) e^{-ik\eta} \right\}$$

Which solution (or state)?

The general solution again,

$$\varphi_k = \frac{H}{\sqrt{2k^3}} \frac{1}{\sqrt{1 - f_k f_k^\star}} \left\{ (i - k\eta) e^{ik\eta} - f_k (i + k\eta) e^{-ik\eta} \right\}$$

Assume space is flat at small scales: $f_k = 0$

$$\varphi_k(\eta) = \frac{H}{\sqrt{2k^3}} (i - k\eta) e^{ik\eta}$$

Some worries: small means $k/a(\eta) >> H$

1) For $H \sim 10^{15}$ GeV, $k/a(\eta) \rightarrow M_{\rm pl}$?

2) $k/a(\eta) >> H$ depends on the time

The standard prediction

What is the shape of the primordial noise?

$$\langle 0 \,|\, \varphi(\eta,\mathbf{x})\,\varphi(\eta,\mathbf{y}) \,|\, 0 \rangle = \int \frac{d^3\mathbf{k}}{(2\pi)^3}\, e^{i\mathbf{k}\cdot(\mathbf{x}-\mathbf{y})}\, \frac{2\pi^2}{k^3}\, P_k(\eta)$$

The power spectrum,

$$P_k(\eta) = \frac{k^3}{2\pi^2}\,\varphi_k\varphi_k^{\star} = \frac{H^2}{4\pi^2}\,(1 + k^2\eta^2)$$

By the end of inflation, the modes have been stretched well outside the horizon,

$$k << a(\eta)H = \frac{H}{-H\eta} \quad\Rightarrow\quad -k\eta << 1$$

The power spectrum is flat: $P_k \approx H^2/4\pi^2$

Finer points

To be more realistic:

1) The background fluctuates too

$$a^2(\eta)\ \eta_{\mu\nu} \rightarrow g_{\mu\nu} = a^2(\eta)\ [\ \eta_{\mu\nu} + \delta g_{\mu\nu}(\eta,\mathbf{x})\ ]$$

2) φ = "inflaton + scalar part of metric"

3) there are tensors too, $h_{ij}(\eta,\mathbf{x}) \in \delta g_{ij}$

so we are actually considering *quantum fluctuations of gravity*

which is reasonable as long as we are not in the Planck regime

Which wavenumbers (k) are safe?

At a minimum, k_{min}

A mode entering the horizon today was just leaving the horizon when we start our evolution, say at η_0,

$$\frac{k_{\text{min}}}{a(\eta_0)} = H \quad \Rightarrow \quad k_{\text{min}} = -\frac{1}{\eta_0}$$

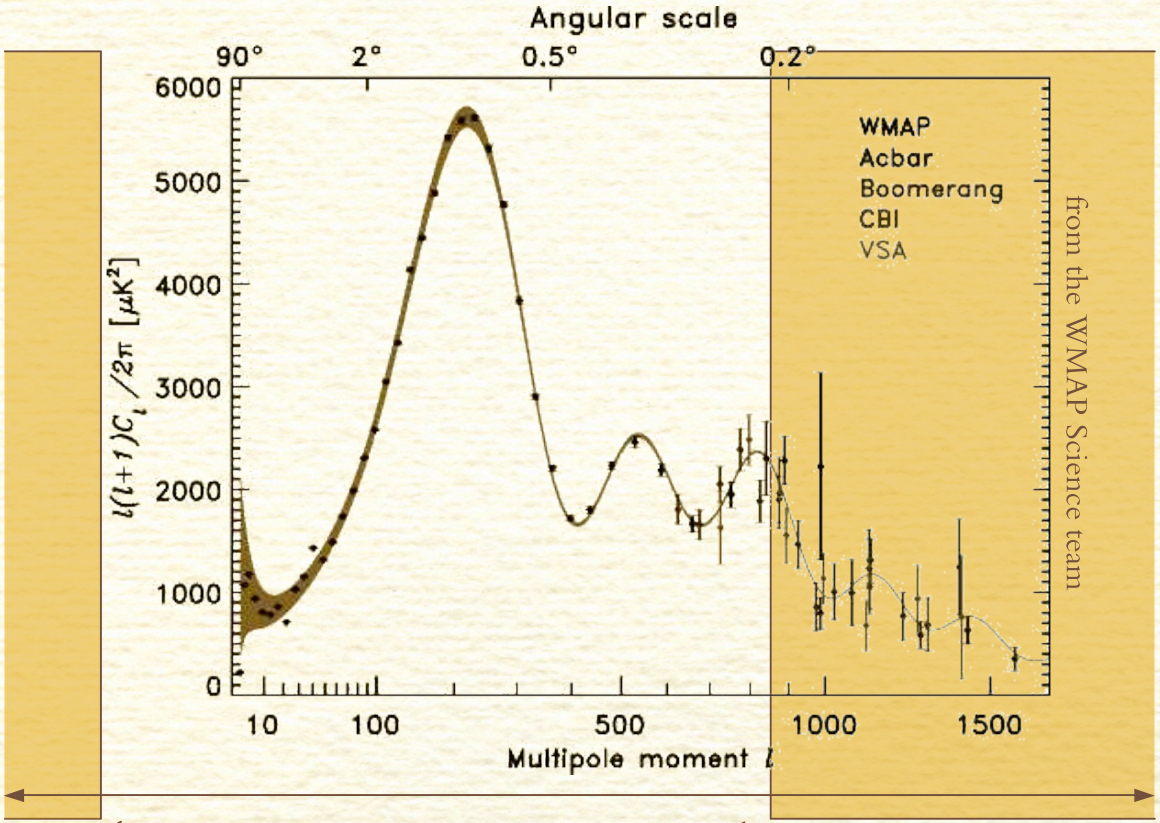
At the maximum, $k_\star$

no “trans-Planckian” modes at η_0,

$$\frac{k_\star}{a(\eta_0)} = M_{\text{pl}} \quad \Rightarrow \quad k_\star = -\frac{M_{\text{pl}}}{H\eta_0} = \frac{M_{\text{pl}}}{H} k_{\text{min}}$$

So $$\frac{H}{M_{\text{pl}}} k_\star < k < k_\star$$

Which wavenumbers (k) are safe?



$k_{\rm min}$ $k_\star$ "trans-Planckian"

For $H \sim 10^{15}$ GeV, $\dfrac{k_\star}{k_{\rm min}} = \dfrac{M_{\rm pl}}{H} \sim 10^3$

How to approach the problem

1) Assume that we know what happens beyond the Planck scale

(historically, this was the first approach)

2) Assume that beyond the Planck scale, space-time still looks "flat"

"adiabatic states"
Mottola, Molina-Paris, Anderson

3) Look for general descriptions for the effects from beyond the Planck scale

effective theory approaches
Collins & Holman
Schalm, Shiu, van der Shaar

A simple framework

To model some of the odd things that could happen, add some *symmetry-breaking operators*,

$$L_S = \frac{{c_S}^{p,q}}{a^q M^{p+q-2}} H^p \, \varphi(-\vec{\nabla}\cdot\vec{\nabla})^{q/2}\varphi$$

$$L_T = \frac{{c_T}^{p,q}}{a^q M^{p+q-2}} H^p \, \boldsymbol{\tau}_{ij}(-\vec{\nabla}\cdot\vec{\nabla})^{q/2}\boldsymbol{\tau}^{ij}$$

These modify the scalar (φ) and tensor ($\boldsymbol{\tau}_{ij}$) power spectra, *e.g.* [for q = 1, 2, >2]

$$P_k{}^{\tau} = \frac{H^2}{\pi^2} + \frac{H^2}{\pi^2}\frac{H^{p+q-2}}{M^{p+q-2}} \begin{cases} 2\pi {c_T}^{p,1} + \cdots \\ 2{c_T}^{p,2}\left[3 + \cos\left[2\frac{M}{H}\frac{k}{k_\star}\right]\right] + \cdots \\ {c_T}^{p,q}\left[\frac{M}{H}\frac{k}{k_\star}\right]^{q-2}\cos\left[2\frac{M}{H}\frac{k}{k_\star}\right] + \cdots \end{cases}$$

The scalar to tensor ratio

Define the scalar to tensor ratio (r) by

$$r = 4\varepsilon \frac{P^{\tau}(k_0\eta)}{P^{\varphi}(k_0\eta)}$$

ε is a slow-roll parameter, $\varepsilon = -H'/(aH^2)$

A few quick observations:

1) a constant shift, $r = 16\varepsilon \dfrac{1 + 2\pi c_T}{1 + \pi c_S}$

2) oscillations, $r = 16\varepsilon \left[1 + \dfrac{k_0 c_S}{2k_\star} \cos\left(2\dfrac{M}{H}\dfrac{k_0}{k_\star}\right)\right]^{-1}$

3) living in a trough (*i.e.*, does k_0 matter?)

Conclusions

Possibilities:

1) The details of the vacuum at trans-Planckian lengths are important and must be included in some way

   very high energies might be experimentally accessible

2) There is some principle that selects the "standard" vacuum

   and it must be imposed on any quantum description of gravity

3) Something is wrong about our standard inflationary picture

the end