The Trispectrum of Curvature Perturbations in Single-field Inflation

Philip R. Jarnhus

CAPPA, University of Aarhus

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Introduction

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- Provides an explanation for the CMB homogeneity and a source for the fluctuation in the CMB
- Details of inflation are not known at present time

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- Data will soon be good enough to determine the level of non-gaussianity in the fluctuations
- In single-field models this gives indications about the interactions of the model

Gauge

- We choose the uniform density gauge and neglect tensor perturbations

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- Constructed such that each spacetime slice has the same energy density all over
- The inflaton field (ϕ_c) is a classic scalar field without quantum fluctuations in this gauge

Spectra

- We define the power spectrum from the two-point function

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- One can now write the bi- and tri-spectrum in terms of the power spectrum

$$B = -\frac{6}{5} f_{NL} [\mathcal{P}(k_1)\mathcal{P}(k_2) + 2 \text{ permutations}]$$

$$T = \frac{1}{2} \tau_{NL} [\mathcal{P}(k_1)\mathcal{P}(k_2)\mathcal{P}(k_{14}) + 23 \text{ permutations}] \ , \ \mathbf{k}_{ij} = \mathbf{k}_i + \mathbf{k}_j$$

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- WMAP five-year data constrains the non-linearity parameter f_{NL} to $-151 < f_{NL} < 253$ (95% CL) [Komatsu et. al. (2008), arXiv:0803.0547]

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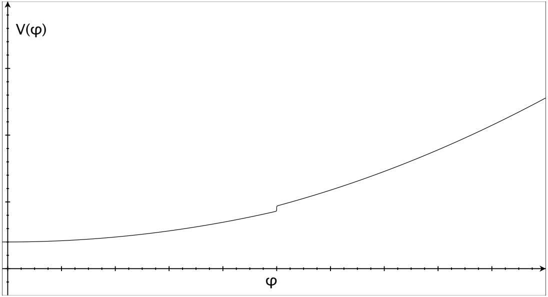
- For a given potential one can now calculate bi- and tri-spectrum, using the appropriate order Hamiltonian

3rd order: [J. Maldacena (2002), arXIv:astro-ph/0210603] , 4th order: [P.R.J. & Martin S. Sloth, arXiv: 0709.2708]

Potential



Collaboration with Steen Hannestad, Troels Haugbølle and Martin S. Sloth

- Potential with small step $V(\phi) = \frac{1}{2}m^2\phi^2\left(1 + c\tanh\left(\frac{\phi - \phi_{step}}{d}\right)\right)$

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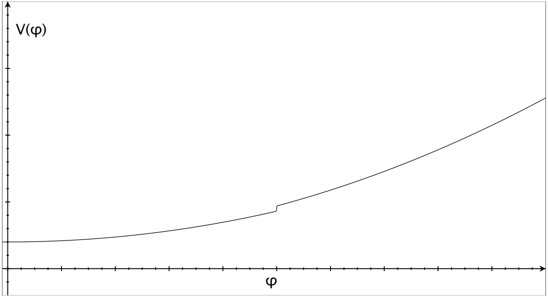

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- Potential with small step $V(\phi) = \frac{1}{2}m^2\phi^2\left(1 + c\tanh\left(\frac{\phi - \phi_{step}}{d}\right)\right)$
- Corresponds to having the inflaton field undergo a phase transition

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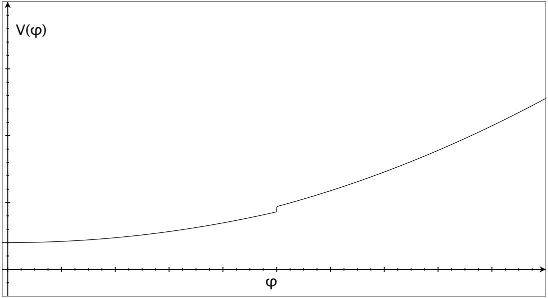

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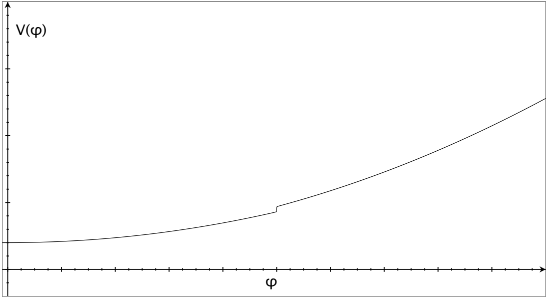

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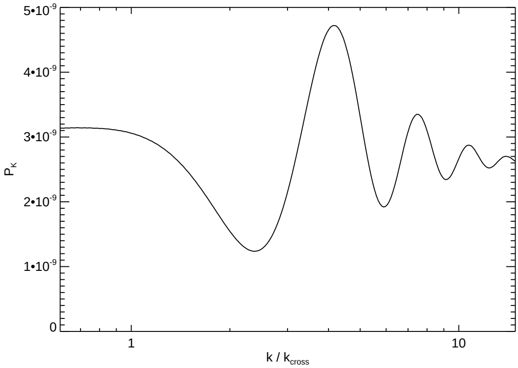
- η and its derivatives becomes large in vicinity of the step
- Leaves an easily recognisable signature in the spectra

Results - Power spectrum



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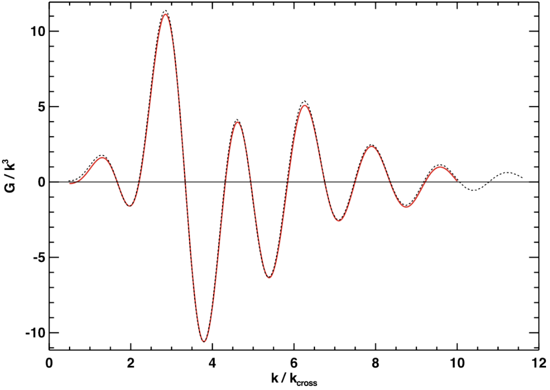
- With parameters $c = 0.0018, d = 0.022M_p, \phi_{step}14.84M_p$
- Power spectrum constrains parameters

Results - Bispectrum (Teaser)



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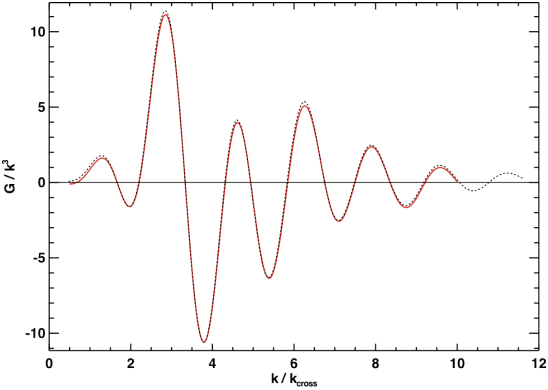
- $\frac{G}{k^3} \propto \frac{\langle \zeta_k^3 \rangle}{P(k)^2}$. First done by Chen et. al (arXiv:0801.3295)
- Step gives large oscillation in bispectrum

Results - Bispectrum (Teaser)



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- Trispectrum is expected to have similar features as bispectrum

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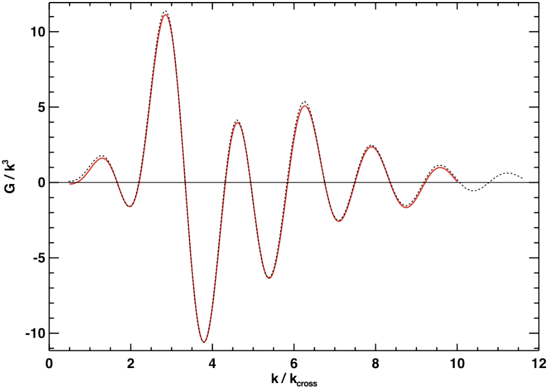

Collaboration with Steen Hannestad, Troels Haugbølle and Martin S. Sloth

- Trispectrum is expected to have similar features as bispectrum
- We plan to have a trispectrum before long

Conclusions

- The bi- and trispectrum provides an insight into the model governing inflation
- Steps in the potential leaves significant imprints on the bi- and trispectrum
- Future experiments (Planck) will be able to detect or constrain the size of the step

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ADVERTISEMENT
The working code will be made available to the public in the near future.
It provides an easy way to calculate powerspectrum, as well as the bi- and trispectrum for single-field infaltion in the uniform density gauge