Multi-field DBI inflation

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Based on David Langlois, S. R-P, arXiv:0801.1085 [hep-th], JCAP
David Langlois, S. R-P, D. Steer & T. Tanaka, arXiv:0804.3139 [hep-th], PRL
David Langlois, S. R-P, D. Steer & T. Tanaka, arXiv:0806.0336 [hep-th], PRD

UniverseNet, Oxford, 24.09.08

Outline

I : Action and background

II : Linear perturbations

III : Non-Gaussianities

Introduction

- After 25 years of existence, inflation has been so far very successful to account for observational data.

- Many high energy physics models involve several scalar fields. If several scalar fields are light enough during inflation ⇨ **multi-field inflation** !

- Models with **non-standard** kinetic terms: k-inflation, DBI inflation, …

- Multi-field extension and perturbations ?

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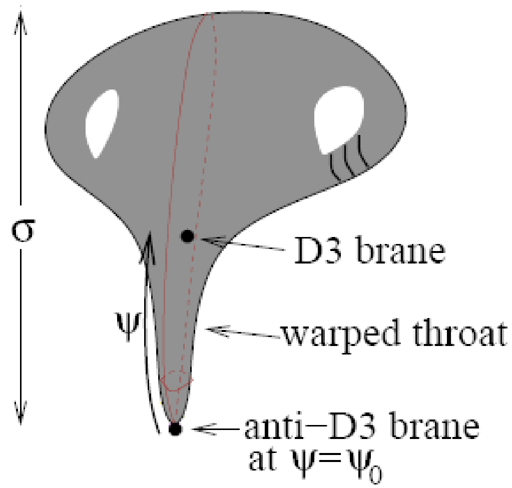
DBI inflation



– Brane inflation: inflaton as the distance between two branes

– Moving D3-brane in a higher-dimensional background

$$ds^2 = h^{-1/2}(y^K) g_{\mu\nu} dx^\mu dx^\nu + h^{1/2}(y^K) G_{IJ}(y^K) dy^I dy^J$$

Aim : take into account all the internal coordinates

➡ Multi-field effective description [Easson et al. '07; Huang et al. '07]

The dynamics is governed by a Dirac-Born-Infeld action

$$L_{DBI} = -\frac{1}{f}\sqrt{-\det\left(g_{\mu\nu} + f\, G_{IJ}\, \partial_\mu \phi^I \partial_\nu \phi^J\right)} \qquad f = \frac{h}{T_3}$$

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DBI action

Including potential terms

Action $$S = \int d^4x \sqrt{-g} \left(\frac{M_p^2}{2} R^{(4)} - \frac{1}{f} \left(\sqrt{\mathcal{D}} - 1 \right) - V(\phi^I) \right)$$

with $$\mathcal{D} = \det \left(\delta^\mu_\nu + f G_{IJ} \partial^\mu \phi^I \partial_\nu \phi^J \right)$$

Background : homogeneous fields

$$\mathcal{D} = 1 - f G_{IJ} \dot{\phi}^I \dot{\phi}^J$$

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DBI action

Multiple inhomogeneous fields

Lorentz covariance allows to consider

$$X^{IJ} = -\frac{1}{2}\partial^\mu \phi^I \partial_\mu \phi^J \qquad X_I^J = G_{IK} X^{KJ}$$

$$\mathcal{D} = 1 - 2fG_{IJ}X^{IJ}$$
$$+4f^2 X_I^{[I} X_J^{J]} - 8f^3 X_I^{[I} X_J^J X_K^{K]} + 16 f^4 X_I^{[I} X_J^J X_K^K X_L^{L]}$$

Terms which vanish for

- one field

- multiple homogeneous fields

Essential for perturbations

The object of our work

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A taste of DBI inflation

- In the homogeneous, one field, situation,

$$S = \int \mathrm{d}t a^3 \left[-\frac{1}{f}\left(\sqrt{1 - f\dot{\phi}^2} - 1\right) - V(\phi) \right]$$

1. Slow-roll regime:

$$f\,\dot{\phi}^2 \ll 1 \qquad\qquad S = \int \mathrm{d}t a^3 \left[\frac{1}{2}\dot{\phi}^2 - V(\phi) \right]$$

2. "Relativistic" regime:

$$c_s^2 \equiv 1 - f\,\dot{\phi}^2 \ll 1 \Rightarrow |\dot{\phi}| \simeq 1/\sqrt{f}$$

[Silverstein, Tong '04; Alishahiha, Silverstein, Tong'04]

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A taste of DBI inflation

- How can we achieve inflation ?

$$\epsilon \equiv -\frac{\dot{H}}{H^2} = \frac{3}{2}\frac{1/c_s - c_s}{1/c_s - 1 + fV} = \frac{3}{2}\frac{1/c_s f\dot{\phi}^2}{1/c_s - 1 + fV}$$

1. Slow-roll regime:

$$c_s \simeq 1 \qquad \epsilon \simeq \frac{3}{2}\frac{\dot{\phi}^2}{V} \ll 1$$

for a potential dominated inflation

2. “Relativistic” regime:

$c_s \ll 1$ with $fV \gg 1/c_s$ ⇒ $\epsilon \simeq \frac{3}{2}\frac{1/c_s}{fV} \ll 1$

Linear perturbations : one field

- Second-order action

$$\mathcal{L}_2 = \frac{a^3}{2c_s^3}\left[\dot{Q}_\sigma^2 - c_s^2\left(\frac{1}{a}\nabla Q_\sigma\right)^2\right] + \dots$$

Q_σ: field perturbation in the flat gauge

- Canonically normalized field (in conformal time)

$$v_\sigma = \frac{a}{c_s^{3/2}} Q_\sigma$$

- Equation of motion in the slow-varying regime

$$v_\sigma'' + \left(c_s^2 k^2 - \frac{2}{\tau^2}\right) v_\sigma = 0$$

- Amplification at sound horizon crossing

$$v_\sigma \simeq \frac{1}{\sqrt{2kc_s}} e^{-ikc_s\tau}\left(1 - \frac{i}{kc_s\tau}\right)$$

$$c_s k = aH$$

$$\Rightarrow \quad \mathcal{P}_{Q_\sigma} = \left(\frac{H}{2\pi}\right)^2$$

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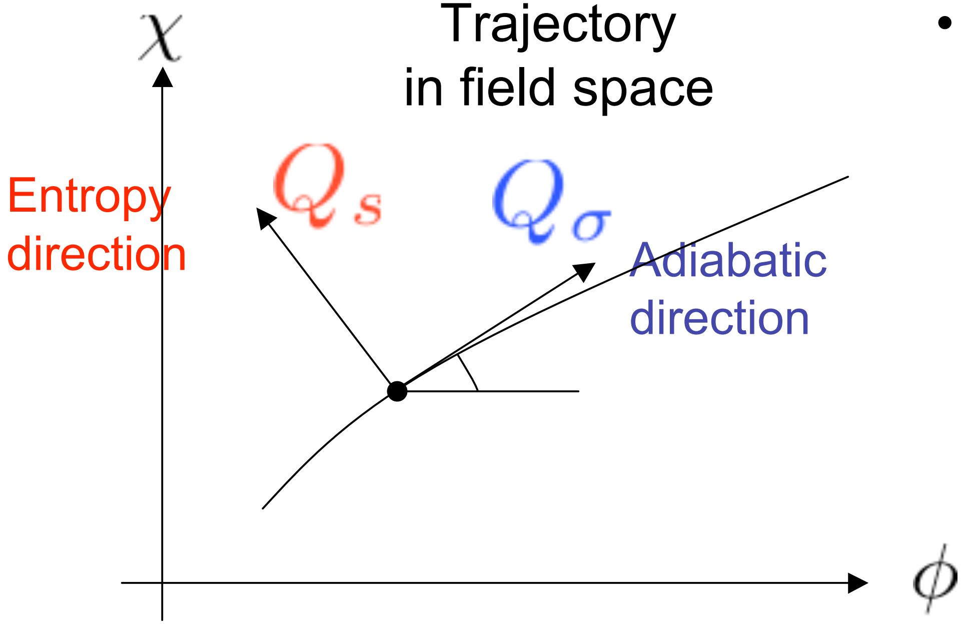
Linear perturbations : two fields

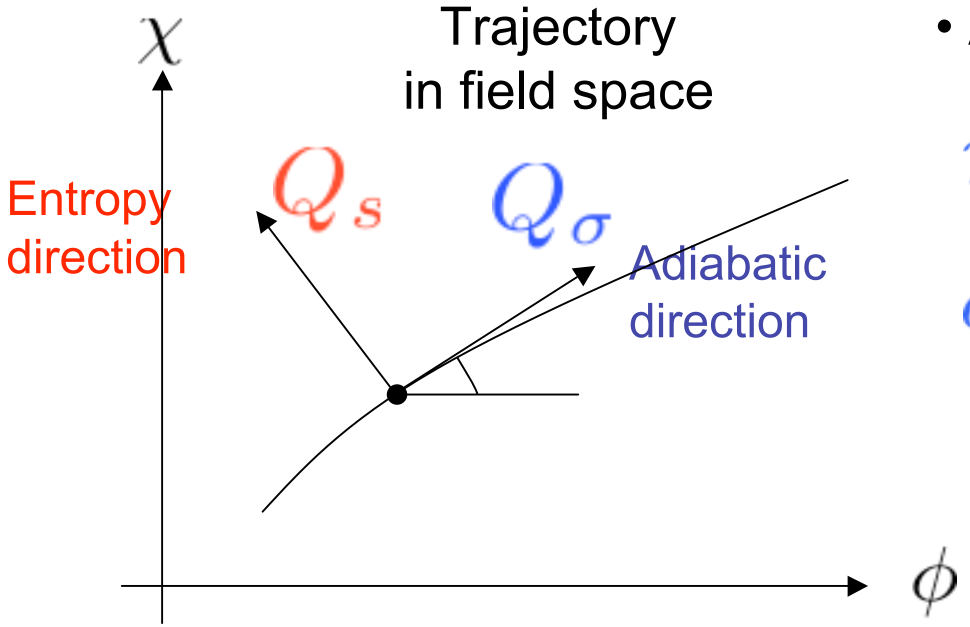


- Adiabatic / Entropy decomposition

$$\mathcal{R} = \frac{H}{\dot{\sigma}} Q_\sigma \qquad \dot{\sigma}^2 = G_{IJ} \dot{\phi}^I \dot{\phi}^J$$

$$c_s^2 = 1 - f \dot{\sigma}^2$$

Linear perturbations : two fields



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- Second-order action:

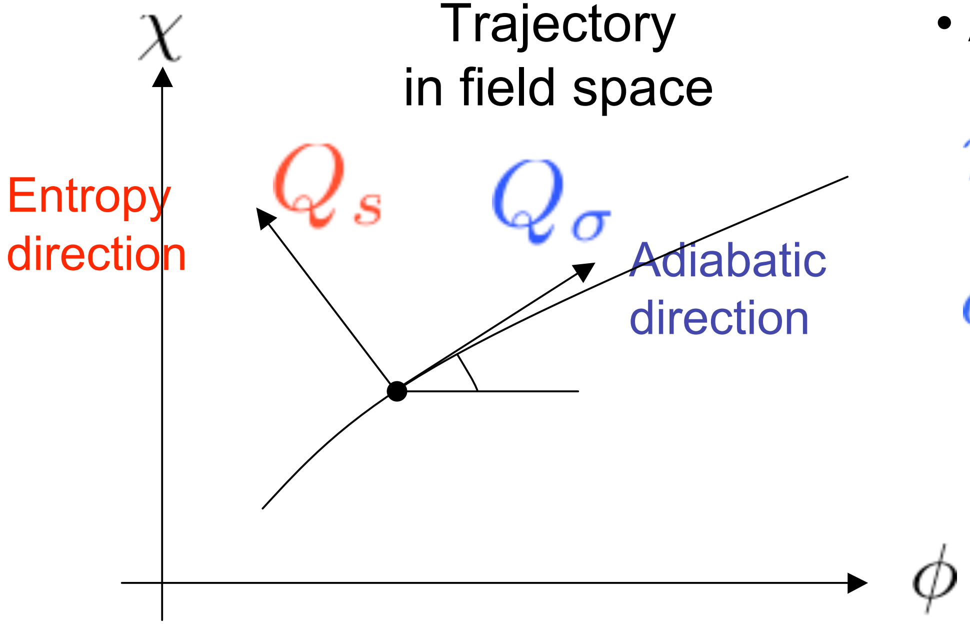
$$\mathcal{L}_2 = \frac{a^3}{2c_s^3}\left[\dot Q_\sigma^2 - c_s^2\left(\frac{1}{a}\nabla Q_\sigma\right)^2\right] + \frac{a^3}{2c_s}\left[\dot Q_s^2 - c_s^2\left(\frac{1}{a}\nabla Q_s\right)^2\right]$$

- Canonically normalized field

$$v_s = \frac{a}{c_s} Q_s \qquad v_\sigma \simeq v_s$$

$$\Rightarrow \quad \mathcal{P}_{Q_\sigma} = \left(\frac{H}{2\pi}\right)^2 \qquad \mathcal{P}_{Q_s} = \left(\frac{H}{2\pi c_s}\right)^2$$

Linear perturbations : two fields



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- Canonically normalized field

$$v_s = \frac{a}{c_s} Q_s \qquad v_\sigma \simeq v_s$$

⇒ Enhancement of isocurvature perturbations

$$Q_s \simeq \frac{1}{c_s} Q_\sigma$$

Primordial spectra

- Curvature perturbation $$\mathcal{P}_{\mathcal{R}_*} = \frac{H^2}{8\pi\epsilon c_s}$$

[same as single-field k-inflation:
Garriga & Mukhanov '99]

- In the multi-field case, $\mathcal{R}$ can evolve on large scales

$$\mathcal{R} = \mathcal{R}_* + T_{\mathcal{RS}}\mathcal{S}_* \quad \left[\mathcal{S} = c_s \frac{H}{\dot{\sigma}} Q_s\right] \quad \mathcal{P}_{\mathcal{R}} = (1 + T_{\mathcal{RS}}^2)\mathcal{P}_{\mathcal{R}_*} = \frac{\mathcal{P}_{\mathcal{R}_*}}{\cos^2\Theta}$$

Feeding of curvature perturbation by entropy perturbations

- Tensor modes

$$\mathcal{P}_{\mathcal{T}} = \left(\frac{2H^2}{\pi^2}\right)_{k=aH} \quad \Rightarrow \quad r = \frac{\mathcal{P}_{\mathcal{T}}}{\mathcal{P}_{\mathcal{R}}} = 16\epsilon\, c_s \cos^2\Theta$$

Non-Gaussianities

$$\langle \mathcal{R}(\boldsymbol{k}_1)\mathcal{R}(\boldsymbol{k}_2)\mathcal{R}(\boldsymbol{k}_3)\rangle = -(2\pi)^7\delta(\sum_i \boldsymbol{k}_i)(\frac{3}{10}f_{NL}(\mathcal{P}_\mathcal{R})^2)\frac{\sum_i k_i^3}{\prod_i k_i^3}$$

Slow-roll inflation $f_{\rm NL} \simeq 10^{-2}$

Planck accuracy: $\Delta f_{\rm NL} \simeq 5$

Single-field DBI $f_{\rm NL}^{(equil)} = -\frac{35}{108}\frac{1}{c_s^2}$

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Non-Gaussianities : influence of isocurvature perturbations

- Third order action

$$S_3^{(\mathrm{main})} = \int dt d^3x \left\{ \frac{a^3}{2c_s^5\dot{\sigma}} \left[\dot{Q}_\sigma^3 + c_s^2 \dot{Q}_\sigma \dot{Q}_s^2 \right] - \frac{a}{2c_s^3\dot{\sigma}} \left[\dot{Q}_\sigma (\nabla Q_\sigma)^2 + c_s^2 \dot{Q}_\sigma (\nabla Q_s)^2 - 2c_s^2 \dot{Q}_s \nabla Q_\sigma \nabla Q_s \right] \right\}$$

- Shape of $f_{\rm NL}$ unaltered

- Amplitude $f_{NL}^{(\mathrm{equil})} = -\frac{35}{108}\frac{1}{c_s^2}\frac{1}{1+T_{\mathcal{RS}}^2} = -\frac{35}{108}\frac{1}{c_s^2}\cos^2\Theta$

Non-Linearity parameter
reduced by entropy
perturbations

Very important for
model-building

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Conclusions

- Angular motion of the D3-brane have important observational consequences:
- enhancement of isocurvature perturbations with respect to the adiabatic one.
- amplitude of non-Gaussianities reduced.

- General analysis of the perturbations in multi-field models with non-canonical kinetic terms

$$P(-\frac{1}{2}\partial_\mu\phi\partial^\mu\phi,\phi) \quad \Rightarrow \quad P(X^{IJ},\phi^K)$$

  – Multi-field extension of k-inflation
  – Particular case: multi-field DBI inflation

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