A prior dependence of the tensor-to-scalar ratio

Wessel Valkenburg

Laboratoire d'Annecy-Le-Vieux de Physique Théorique

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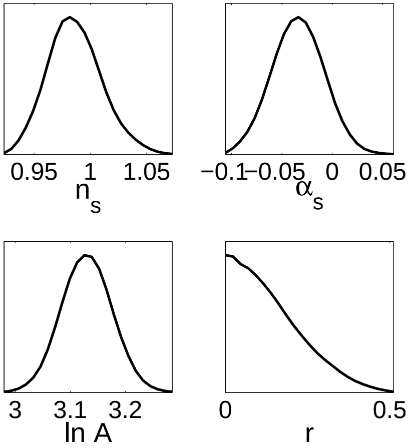

$+\ \theta, \tau, \Omega_{\rm dm}, \Omega_{\rm b}$
(using WMAP5 + SDSS-LRG DR4)

Inflationary history

$$H(\phi) = H_* + (\phi - \phi_*) H'_* + \frac{1}{2}(\phi - \phi_*)^2 H''_* + \frac{1}{6}(\phi - \phi_*)^3 H''' + \ldots$$

$$H' \equiv \partial_\phi H$$

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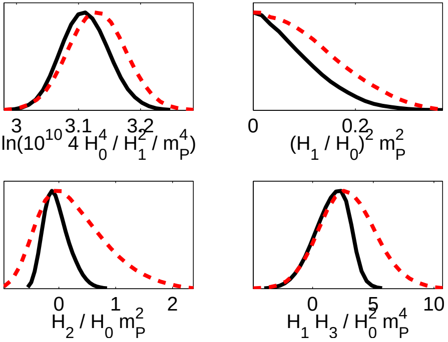

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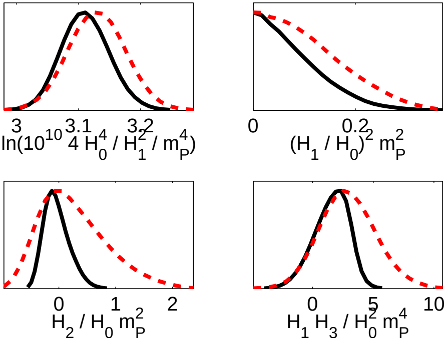

$$\ln A_S \sim \ln \left[\frac{4 H_*^4}{H_*'^2 m_P^4} 10^{10} \right]$$

$$n_S \sim \left(\frac{H'_*}{H_*} \right)^2 m_P^2, \; \frac{H''_*}{H_*} m_P^2$$

$$r \sim \left(\frac{H'_*}{H_*} \right)^2$$

$$\alpha_S \sim \frac{H'_* H'''_*}{H_*^2} m_P^4$$

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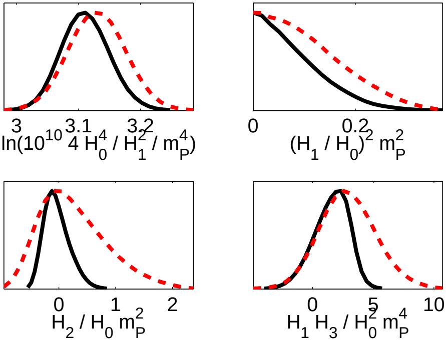

Flat prior on

$$\ln\left[\frac{4H_*^4}{H_*'^2 m_P^4} 10^{10}\right],$$

$$\left(\frac{H'_*}{H_*}\right)^2 m_P^2,\ \frac{H''_*}{H_*} m_P^2,$$

$$\frac{H'_* H'''_*}{H_*^2} m_P^4$$

Flat prior on $\ln A_S$, n_S, α_S, r

[Lesgourgues, Starobinsy, WV, 2007]

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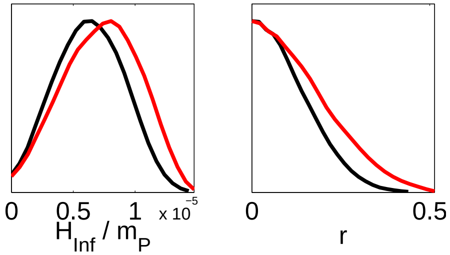

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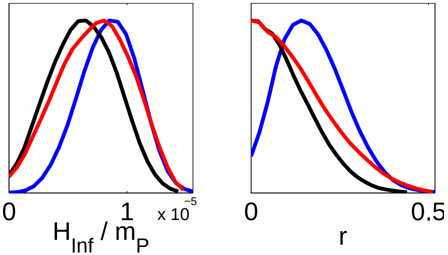

[WV, Krauss, Hamann, 2008]

Flat prior on
$\ln\left[\frac{4H_*^4}{H_*'^2 m_P^4}10^{10}\right]$,
$\left(\frac{H'_*}{H_*}\right)^2 m_P^2$, $\frac{H''_*}{H_*} m_P^2$,
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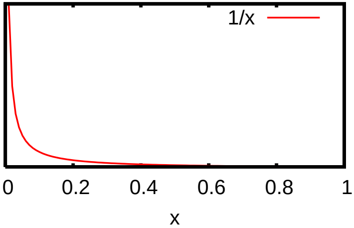
Flat prior on $\ln A_{\rm S}$, $n_{\rm S}$, $\alpha_{\rm S}$, r

Flat prior on $H_{\rm inf}$, H', H'', H'''

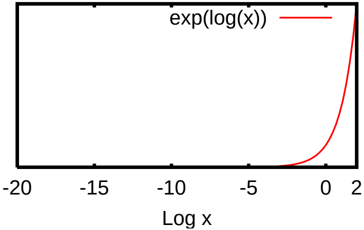
$\log x$ versus x

$$x \in [0,1] \Longleftrightarrow \log x \in [-\infty, 0]$$

Prior on x under a flat prior on Log x

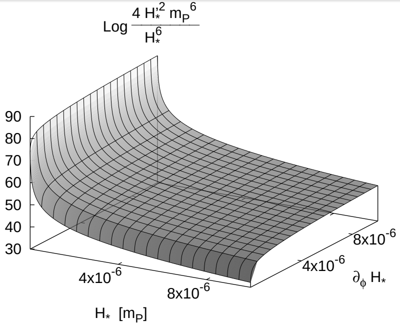


Prior on Log x under a flat prior on x



$$\text{Posterior} = \text{Prior} \times \text{Likelihood}$$

$$\text{Flat prior on } \left\{ \ln\left[\frac{4H_*^4}{H_*'^2 m_P^4} 10^{10}\right], \left(\frac{H_*'}{H_*}\right)^2 m_P^2 \right\} \text{ versus } \{H_{\rm inf}, H'\}$$



$$\text{Posterior} = \text{Prior} \times \text{Likelihood}$$

$$\text{Prior}_{\{x_i\}} = \left|\frac{dy_i}{dx_j}\right| \text{Prior}_{\{y_i\}}$$

Inflationary history

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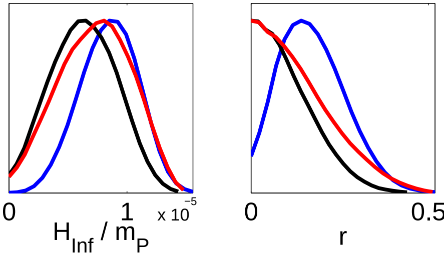

[WV, Krauss, Hamann, 2008]

$$+\ \theta, \tau, \Omega_{\rm dm}, \Omega_{\rm b}$$

Flat prior on $\ln\left[\frac{4H_*^4}{H_*'^2 m_P^4} 10^{10}\right]$, $\left(\frac{H'_*}{H_*}\right)^2 m_P^2$, $\frac{H''_*}{H_*} m_P^2$, $\frac{H'_* H'''_*}{H_*^2} m_P^4$

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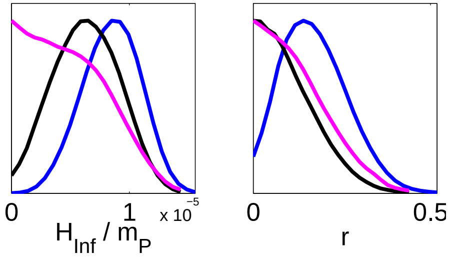

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Mean likelihood

[WV, Krauss, Hamann, 2008]

Flat prior on set:	r (at 68% c.l.)	$C_{\rm B}$
$\{a_i\}$	$r < 0.15$	6.98 ± 0.03
$\{b_i\}$	$r < 0.04$	6.25 ± 0.8
$\{c_i\}$	$0.061 < r < 0.243$	7.80 ± 0.03
$\{d_i\}$	$r < 0.2$	7.45 ± 0.05

$$\{a_i\} = \{A_{\rm S}, \frac{H'^2}{H^2} m_{\rm P}^2, \frac{H''}{H} m_{\rm P}^2, \frac{H' H'''}{H^2} m_{\rm P}^4\},$$

$$\{b_i\} = \{A_{\rm S}, \ln \frac{H'^2}{H^2} m_{\rm P}^2, \frac{H''}{H} m_{\rm P}^2, \frac{H' H'''}{H^2} m_{\rm P}^4\},$$

$$\{c_i\} = \{H, H', H'', H'''\},$$

$$\{d_i\} = \{\ln H, H', H'', H'''\},$$

$$C_{\rm B} \equiv \overline{\chi^2} - \chi^2(\hat{\theta})$$

[Trotta, 2008]

Conclusion

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Conclusion

- If the Bayesian Complexity C_b is less than the number of parameters, it is likely that your parameter estimates have a prior dependence.
- As a consequence, the present upper bound on the tensor-to-scalar ratio is not as well defined as it looks.
- And most importantly ....
- .... that's all.