DARK MATTER AND DARK ENERGY FROM LAGRANGIAN WITH CANONICAL KINETIC TERMS

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ABSTRACT AND MOTIVATION

In the context of the k-essence field, a lagrangian (incorporating Scherrer's scaling relation) is set up with canonical kinetic terms.This lagrangian can account for both dark matter and dark energy.

Presence of canonical kinetic terms ensure that the fundamentals of lagrangian field theory can be used effectively to understand cosmological scenarios.

Cosmological quantities calculated from this lagrangian give reasonably good results and certain aspects of the early universe can also be studied in this formalism.

Early Universe corresponds to small values of the scale factor and this can be tackled in a standard lagrangian formalism.

LAGRANGIAN AND SCALING RELATION

A lagrangian **L** for the k- essence field **φ** is set up incorporating the scaling relation of Scherrer :

Lagrangian:

- $$L = - V(\Phi)\, F(X) , \qquad X = \nabla_\mu \Phi \nabla^\mu \Phi$$

Scherrer's scaling relation:

- $$X F_x^{\,2} = c\; a^{-6} , \qquad F_X = dF/Dx$$

a is the scale factor and **c** is a constant

THE COSMOLOGICAL PARAMETERS

Energy Density : $\rho = V(\Phi)\,[\,F(X) - 2\,X\,F(X)]$

Equation of state parameter : $w = p / \rho$

Sound velocity : $c_s^{\,2} = \partial p / \partial \rho$

Deceleration parameter : $Q = a\,(d^2 a/dt^2)/(da/dt)^2$

F(X) is a function of **X** , **Φ** a scalar field , **c** is a constant.

THE LAGRANGIAN WITH CANONICAL KINETIC TERMS

Taking the zero-zero component of Einstein field equations:

- $R_{00} - (1/2)\, g_{00}\, R = - \kappa\, T_{00}$

And the Robertson –Walker metric :

- $ds^2 = c^2\, dt^{\,2} - a^2(t)\, [\, dr^2/(1 - k\, r^2)$

  $+ r^2\, (\, d\,\theta^2 + \sin^2\theta\, d\,\Phi^{\,2})]\,,$

With **k=0**, and incorporating Scherrer's scaling relation, and using homogeneity and isotropy of space so that $\Phi(x,t) = \Phi(t)$

The lagrangian takes the form :

- $L = - c_1\, (dq/dt)^2 - c_2\, V(\Phi)\, (d\,\Phi/dt)\, e^{-3q}\,,$

  $c_1 = 3(8\pi\, G)^{-1}\,, \qquad c_2 = 2\, c^{1/2}$

The two degrees freedom are $q(t) = \ln a(t)$ and φ

THE EQUATIONS OF MOTION

The equations of motion obtained for the two degrees freedom **$q(t) = \ln a(t)$** and **φ** are

- $(d/dt)\,[2\,c_1\,(dq/dt)] = -\,3c_2\;V(\Phi)\,(d\,\Phi/dt)\,e^{-3q}$
- $(d\,\Phi/dt) = -\,[\,3\,V(\Phi)\,(dq/dt)]\;/\;(\partial V\,/\,\partial\Phi)$

To solve these equations we assume

- $V^{2}(\Phi)\,/\,(d\,V/d\,\Phi) = -\,A_1$

where A_1 is a positive constant.

THE SOLUTIONS

The classical solutions are

Scale factor

$$a_c\,(t) = [\,e^{\alpha t} - \beta\,]^{1/3}$$

Scalar Field

$$\varphi_c\,(t) = e^{\alpha t} - \beta - A_2$$

Hubble Parameter

$$H_c = \alpha\, e^{\alpha t} \;/\; 3(\,e^{\alpha t} - \beta\,),$$

Scalar field potential

$$V_c = A_1 \,/\, (e^{\alpha t} - \beta\,)$$

Using these solutions quantities of cosmological interest are determined.

THE SOLUTIONS (contd.)

Energy Density:

- $\rho_c = (\alpha^2/ 24\pi G) + (\alpha^2 \beta/ 24\pi G)\, a_c^{-3} (1 - \beta e^{-\alpha t})^{-1}$

$$= (\alpha^2/ 24\pi G) + (\alpha^2 \beta/ 12\pi G)\, a_c^{-3}$$

$$+ (\alpha^2 / 24\pi G)\, a_c^{-3} [\beta^2 e^{-\alpha t} + (\beta^3/2!)\, e^{-2\alpha t} + (\beta^4/3!)\, e^{-3\alpha t} + \ldots..]$$

$$\equiv \rho_{DE} + \rho_{DM} + \rho'$$

$$\rho_{DE} = (\alpha^2/ 24\pi G)$$

$$\rho_{DM} = (\alpha^2/ 12\pi G)\, a_c^{-3}$$

$$\rho' = (\alpha^2 / 24\pi G)\, a_c^{-3} [\beta^2 e^{-\alpha t} + (\beta^3/2!)\, e^{-2\alpha t} + (\beta^4/3!)\, e^{-3\alpha t} + \ldots..]$$

Lagrangian (pressure):

- $p_c = - \rho_c + 2\, c^{1/2} A_1 (1 - \beta e^{-\alpha t})^{-1} a_c^{-3}$

THE SOLUTIONS (contd.)

Equation of state parameter :

- $w = p_c \ / \ \rho_c = -1 + 2\beta e^{-\alpha t}$

Sound Velocity :

- $c_s{}^2 = \delta p / \delta\rho = \beta e^{-\alpha t}$

Deceleration Parameter:

- $Q = a (d^2 a/dt^2)/(da/dt)^2 = -1 + 3\beta e^{-\alpha t}$

RESULTS FOR COSMOLOGICAL PARAMETERS

- The energy density **ρ** has a constant component which we identify as dark energy and a component behaving as **a^{-3}** which we call dark matter.
- The pressure **p** is negative for time **$t \to \infty$**
- The sound velocity **$c_s^2 = \delta p / \delta\rho << 1$.**
- When dark energy dominates ,the deceleration parameter **$Q \to -1$** while in the matter dominated era **$Q \sim 1/2$** .
- The equation of state parameter **$w = p / \rho$** is consistent with **$w \sim -1$** for dark energy domination. During the matter dominated era we have **$w \sim 0$.**
- Bounds for the parameters of the theory are estimated from observational data.

CONCLUSION

Therefore the classical solutions give very encouraging results . A quantum version of the lagrangian has been worked out for applications to early universe .

- *(REF: DG & SM , Phys.Lett. **B665** (2008) 121.)*