Neutrino propagation in the case of general interaction

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Motivations

Let us begin with the process of the production (P) of the massive neutrino ν_i $(i = 1, 2, 3)$ accompanied by the lepton l_α $(\alpha = e, \mu, \tau)$:

$$l_\alpha + P_1 \to \nu_i + P_2 \, , \tag{1}$$

followed, after travelling along the baseline L, by its detection (D):

$$\nu_i + D_1 \to l_\beta + D_2 \tag{2}$$

where P_1, P_2 and D_1, D_2 are the accompanied particles.

1. The effective model Lagrangian

We assume that the interaction is characterized by the effective model Lagrangian. The charged current (CC) Lagrangian is:

$$\begin{aligned}\mathcal{L}_{CC} = \frac{-e}{2\sqrt{2}sin\theta_W}\Big\{ & \textstyle\sum_{\alpha,i} \bar{\nu}_i \left[\gamma^\mu(1-\gamma_5)\varepsilon_L U^{L*}_{\alpha i} + \gamma^\mu(1+\gamma_5)\varepsilon_R U^{R*}_{\alpha i} \right] l_\alpha \, W^+_\mu \\ & + \textstyle\sum_{\alpha,i} \bar{\nu}_i \left[(1-\gamma_5)\eta_L V^{L*}_{\alpha i} + (1+\gamma_5)\eta_R V^{R*}_{\alpha i} \right] l_\alpha \; H^+ \\ & + \textstyle\sum_{u,d} \bar{u} \left[\gamma^\mu(1-\gamma_5)\epsilon^q_L U^*_{ud} + \gamma^\mu(1+\gamma_5)\epsilon^q_R U^*_{ud} \right] d \; W^+_\mu \\ & + \textstyle\sum_{u,d} \bar{u} \left[(1-\gamma_5)\tau_L W^{L*}_{ud} + (1+\gamma_5)\tau_R W^{R*}_{ud} \right] d \; H^+ \Big\} + h.c.\end{aligned} \tag{3}$$

and the neutral current Lagrangian:

$$\begin{aligned}\mathcal{L}_{NC} = -\frac{e}{4sin\theta_W cos\theta_W}\Big\{ & \textstyle\sum_{i,j} \bar{\nu}_i \left[\gamma^\mu(1-\gamma_5)\varepsilon^{N\nu}_L \delta_{ij} + \gamma^\mu(1+\gamma_5)\varepsilon^{N\nu}_R \Omega^R_{ij} \right] \nu_j \; Z_\mu \\ & + \textstyle\sum_{i,j} \bar{\nu}_i \left[(1-\gamma_5)\eta^{N\nu}_L \Omega^{NL}_{ij} + (1+\gamma_5)\eta^{N\nu}_R \Omega^{NR}_{ij} \right] \nu_j \; H^0 \\ & + \textstyle\sum_{f=e,u,d} \bar{f} \left[\gamma^\mu(1-\gamma_5)\varepsilon^{Nf}_L + \gamma^\mu(1+\gamma_5)\varepsilon^{Nf}_R \right] f \; Z_\mu \\ & + \textstyle\sum_{f=e,u,d} \bar{f} \left[(1-\gamma_5)\eta^{Nf}_L + (1+\gamma_5)\eta^{Nf}_R \right] f \; H^0 \Big\}\end{aligned} \tag{4}$$

The proposed Lagrangian introduces all possible new effects during the neutrino journey from the production to detection place (as e.g. neutrino mixed initial state, lack of factorization, neutrino helicity flip).

2. The general effective interaction Hamiltonian

The effective low energy four-fermion Hamiltonian resulting from the former charged and neutral interaction Lagrangians has the general form [1]:

$$\mathcal{H}_{eff} = \textstyle\sum_{f=e,p,n} \frac{G_F}{\sqrt{2}} \sum_{i,j} \sum_{a=V,A,T} \left(\bar{\nu}_i \Gamma^a \nu_j \right) \left[\bar{f} \, \Gamma_a \left(g^{ij}_{fa} + \bar{g}^{ij}_{fa} \gamma_5 \right) f \right] \, , \tag{5}$$

We consider 3 massive neutrino states (i=1,2,3) in 2 possible helicity states $(\lambda = \pm 1)$. In the mass - helicity base $|i, \lambda\rangle$ the Hamiltonian $\mathcal{H}_{i,\lambda;\, k,\eta}$ which describes the coherent neutrino scattering inside matter is 6×6 dimensional matrix:

$$\mathcal{H} = \mathcal{M} + \begin{pmatrix} \mathcal{H}_{--} & \mathcal{H}_{-+} \\ \mathcal{H}_{+-} & \mathcal{H}_{++} \end{pmatrix} , \tag{6}$$

where $\mathcal{M}$ is the mass (kinetic) part: $\mathcal{M} = diag(E^0_1, E^0_2, E^0_3, E^0_1, E^0_2, E^0_3)$ and $E^0_i = E_\nu + \frac{m_i^2}{2E_\nu}$, $i = 1, 2, 3$.

Now, to obtain the formulas for the oscillation probabilities we have to resolve the eigenvalue problem for the Hamiltonian $\mathcal{H}$. To perform the task we decompose it into the νSM part and NP part which then is treated as the small perturbation. Hence:

$$\mathcal{H} = \mathcal{H}^{\nu SM} + \mathcal{H}^{NP} = \mathcal{M} + \begin{pmatrix} H^0_{--} & 0 \\ 0 & H^0_{++} \end{pmatrix} + \delta V \, , \tag{7}$$

The perturbation δV is given entirely by NP and is also decomposed into 3×3 δV_{ab} matrices:

$$\delta V \equiv \mathcal{H}^{NP} = \begin{pmatrix} \delta V_{--} & \delta V_{-+} \\ \delta V_{+-} & \delta V_{++} \end{pmatrix} . \tag{8}$$

3. Production, oscillation and detection

For calculations of the density matrix of the neutrino [2] we use the following conditions:

- For the large oscillation baseline L only neutrino which is produced in the forward direction in the CM frame will reach the detector. Therefore the calculations with $\Theta_P = 0$ of the density matrix in CM are enough.
- For the relativistic neutrino: $\rho^{CM}(p_{cm}) = \rho^{LAB}(p_{lab})$

1. The density matrix for the production process is:

$$\varrho^\alpha_P(\lambda, i; \lambda', i') = \frac{1}{N_\alpha} \textstyle\sum_{\lambda_{P_2}\lambda_{P_1}\lambda_\alpha} A^{\alpha \; \lambda;\lambda_{P_2}}_{i \; \lambda_{P_1},\lambda_\alpha}(\vec{p}) \left(A^{\alpha \; \lambda';\lambda_{P_2}}_{i' \; \lambda_{P_1},\lambda_\alpha}(\vec{p}) \right)^* . \tag{9}$$

2. For the evolution of the statistical operator we use the following formula:

$$\rho^\alpha_P(\vec{x} = \vec{0}, t = 0) \to \rho^\alpha_P(\vec{x} = \vec{L}, t = T) = e^{-i(\mathcal{H}T)} \rho^\alpha_P(\vec{x} = \vec{0}, t = 0) \; e^{i(\mathcal{H}T)} . \tag{10}$$

4. The differential cross section

The differential cross section in the LAB frame of the detector for the β neutrino detection (we start with the α neutrino) is given by:

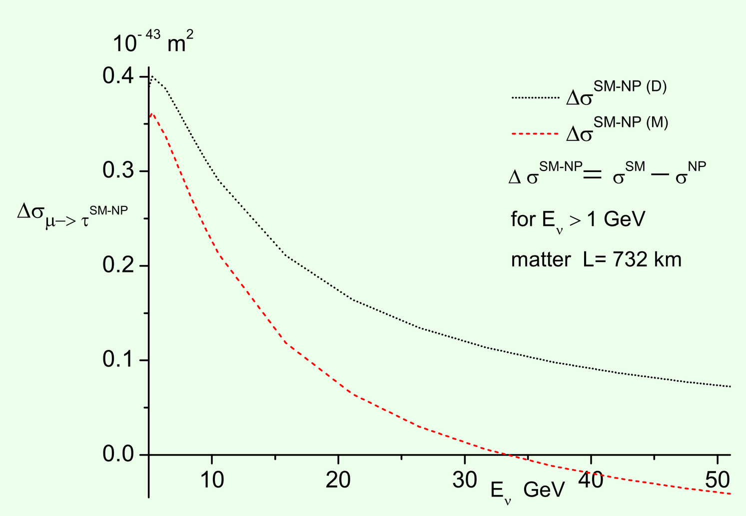
$$\begin{aligned}\frac{d\sigma_{\beta\,\alpha}}{d\Omega_\beta} = & \frac{1}{64\,\pi^2\,(2\,s_{D_1}+1)\;E_\nu\;m_{D_1}} \frac{p^3_\beta}{(E_\nu + m_{D_1})\,p^2_\beta - E_\beta\;(\vec{p}\cdot\vec{p}_\beta)} \\ & \textstyle\sum_{\substack{\lambda,i;\,\lambda',i' \\ \lambda_{D_1},\lambda_{D_2},\lambda_\beta}} A^{\beta \; \lambda,\lambda_{D_1}}_{i \; \lambda_\beta,\lambda_{D_2}}(\vec{p}_\beta) \; \varrho^\alpha_P(\lambda, i; \lambda', i'; L = T) \left(A^{\beta \; \lambda',\lambda_{D_1}}_{i' \; \lambda_\beta,\lambda_{D_2}}(\vec{p}_\beta) \right)^* .\end{aligned} \tag{11}$$

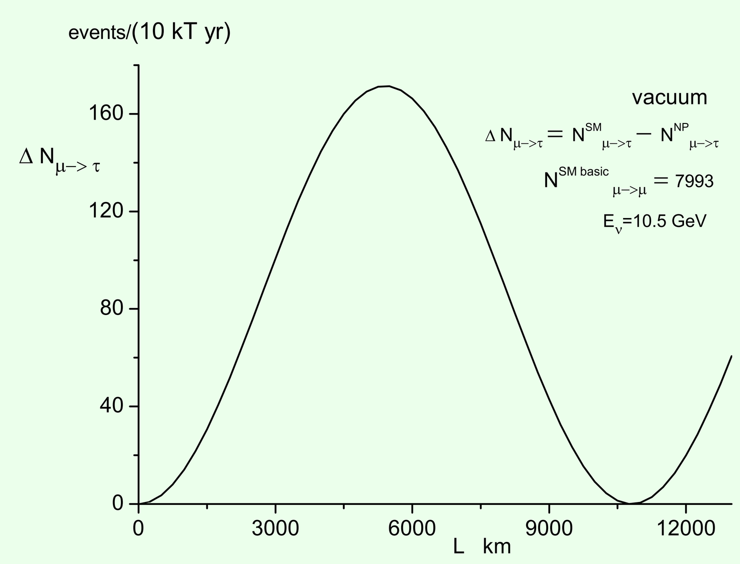
It could be rewritten in the form which after summing over all helicities of the particles is as follows:

$$\begin{aligned}\frac{d\sigma_{\beta\,\alpha}}{d\Omega_\beta} = & \frac{1}{64\,\pi^2\,(2\,s_{D_1}+1)\;E_\nu\;m_{D_1}} \frac{p^3_\beta}{(E_\nu + m_{D_1})\,p^2_\beta - E_\beta\;(\vec{p}\cdot\vec{p}_\beta)} \\ & \textstyle\sum_{i;\,i'} \Big[a^{--}_{\beta;ii'}\,\varrho^\alpha_P(-1, i;\, -1, i'; L) + 2\,cos\varphi\;Re(a^{+-}_{\beta;ii'}\,\varrho^\alpha_P(1, i; -1, i'; L)) \\ & - 2\,sin\varphi\;Im(a^{+-}_{\beta;ii'}\,\varrho^\alpha_P(1, i; -1, i'; L)) + a^{++}_{\beta;ii'}\,\varrho^\alpha_P(1, i; 1, i'; L) \Big] ,\end{aligned}$$

where a-coefficients are the functions of the energies and momenta of the particles in the detection process [3].

5.Some results





6. Comments

- The difference between Dirac and Majorana neutrino is generated by the NP terms.
- The usefulness of the density matrix formalism in the neutrino oscillation analysis follows from the facts:
  1. States are mixed if RH, scalar-LH-RH or pseudoscalar RH-LH interactions are present
  2. Only for relativistic neutrinos produced and detected by the LH mechanism the oscillation rates factorize.

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